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Couette Flow of Two Immiscible Dusty Fluids between Two Parallel Plates with Heat Transfer

[1]W. Abbas, [2]S. M. El Shinnawy, [2]M. S. Emam and [3]Mostafa A. M. Abdeen

[1]Basic and Applied Science Department, College of Engineering and Technology, Arab Academy for Science, Technology, and Maritime Transport, Cairo, Egypt.
[2]Faculty of Engineering (Mataria), Engineering Physics and Mathematics Department, Helwan University, Cairo, Egypt.
[3]Faculty of Engineering, Department of Engineering Mathematics and Physics, Cairo University, Giza, Egypt.

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ABSTRACT

In this paper, the couette flow of two viscous, incompressible, immiscible, electrically conducting dusty fluids between two infinite parallel plates is investigated. The fluids are considered to flow under the effect of pressure gradient. A numerical solution is obtained for the governing non-linear equations by using finite difference method with crank Nicolson technique. Effects of physical parameters such as viscosity ratio, density ratio, conductivity ratio, Eckert and Prandtl number on the velocity and temperature fields for the two fluids and the particles are obtained and represented graphically.

Key words: Immiscible Fluids**,** Heat Transfer, Unsteady flow, Dusty Fluids.

Introduction

The flow and heat transfer of immiscible fluids have special importance in many engineering applications such as the petroleum extraction and transportation lines. So, it has attracted a widespread interest of many scientists. Since the early 1960's, single-phase flow fluids have been studied by many authors, but due to the petroleum industry interest, the flow of two immiscible liquids has been studied. Verma and Bhatt, (1973) carried out one of the first works in the field of two immiscible fluids, they have considered the steady flow of two immiscible incompressible fluids between two infinite parallel plates which the upper plate moves with uniform motion and there is a uniform suction at the stationary lower plate. Sachity, extended the work of Verma and Bhatt to the plane Couette flow of incompressible immiscible second order fluids. Rao and Arayana, (1981) studied a secondary flow and heat transfer of two incompressible immiscible fluids between two parallel plates in a rotating system, the velocity, temperature and skin friction was presented.

Li and Yuriko Renardy, (2000) solved flows of two immiscible liquids at low Reynolds number numerically by using volume-of-fluid (VOF) method and Gauss–Seidel iterative. Sastry *et al*., (2010) discussed a Couette flow of two immiscible fluids between two permeable beds; they found that the velocity is in increasing trend with the increment in the Reynolds number. Flow and heat transfer of two immiscible fluids in the presence of uniform inclined magnetic field have been investigated by Nikodijevic´ *et al.* (2011).

Umavathi *et al.* (2012) analyzed the problem of unsteady mixed convective heat transfer of two immiscible fluids confined between long vertical wavy wall and parallel flat wall, the approximate solutions were obtained by using perturbation techniques, where the dimensionless governing equations are perturbed into a mean part (the zeroth-order) and a perturbed part (the first-order). Champati and Rao, (2013) studied a laminar flow of two immiscible viscous liquids in a saturated porous medium through a rotating straight pipe using the perturbation method.

A study of dispersion of a solute with/without chemical reaction of immiscible electrically conducting fluids between two parallel plates in the presence of an external magnetic field was investigated by Kumar and Umavathi, (2013). A transient Magneto-hydrodynamic flow of two

Corresponding Author: M. S. Emam, Faculty of Engineering (Mataria), Engineering Physics and Mathematics Department, Helwan University, Cairo, Egypt.

immiscible Fluids through a horizontal channel problem has been investigated by Abdul Mateen, (2014). He solved the partial differential equations governing the flow analytically using two-term periodic and non-periodic function in both the regions of the channels and the effect of viscosity ratio, frequency parameter and Hartmann number on the velocity profile.

All the mentioned works, the two fluids are immiscible and pure, but in the present work the fluid is dusty, which makes the equations of our problem are system of coupled partial differential equations and it is hardly to be solved analytically. So, a numerical analysis method is used to solve the problem. Many researchers had done a great work to investigate the dusty fluid, Saffman, (1961) carried out pioneering work on the laminar flow of a dusty gas and introduced the relaxation parameter. He studied the motion of dust particles, and derived the equation of a steady laminar flow. Bo-yi and Osiptsov, (2002) derived a mathematical modeling of near-wall flows of two phase mixture with evaporating droplets. They found that, the combined effect of the droplet evaporation and accumulation results in a significant enhancement of the heat transfer on the surface even for small mass concentration of the droplets in the free stream. Bo-yi *et al.* (2003) studied the flow properties of a dusty-gas point source in a supersonic free stream by using Lagrangian method. They concluded that the sharp accumulation and stratification of the dispersed phase in the shock layers may be of great importance on comet atmospheres and other phenomena. Vetlutskii *et al.* (2005) investigated the problem of gas flow with solid particles in supersonic nozzle using a numerical method. Attia, (2005) presented the Hall Effect on Couette flow with heat transfer of a dusty conducting fluid in the presence of uniform suction and injection. He solved the governing equations numerically using finite differences to yield the velocity and temperature distributions for both fluid and dust particles. Several authors carried out studies of a dusty fluid under different physical conditions (Attia *et al.,* 2015, 2016, 2016; Abdeen *et al.,* 2013).

The objective of the present work is to study two immiscible couette dusty fluid between two infinite parallel horizontal plates. The fluid is considered to be viscous, incompressible and electrically conducting. The governing nonlinear equations for both the fluid and particles are solved numerically by finite difference method. The effects of different flow parameters such as viscosity ratio, conductivity ratio, Eckert and Prandtl number on the velocity and temperature fields for the two fluids and the particles are studied.

Mathematical Formulation:

We consider the unsteady Couette flow of two viscous, incompressible, immiscible fluids between two infinite parallel horizontal plates.

The flow of fluids is separated to two regions as shown in the figure (1). The first region (Region-I) $0 > y > -h$ is filled with a viscous fluid having density ρ_1, dynamic viscosity μ_1, specific heat at constant pressure Cp_1 and thermal conductivity k_1. While the second region (Region-II) is filled with a different viscous fluid having density ρ_2, dynamic viscosity μ_2, specific heat at constant pressure Cp_2 and thermal conductivity k_2. The fluid in the region-I is considered to be a pure fluid while the fluid in region-II has suspended particles. The x axis is taken along the plate in the horizontal direction and the y axis is taken normal to the plates. The fluid properties are considered to be constant, the lower plat is stationary and the upper plat is suddenly moved from rest with velocity U_o.

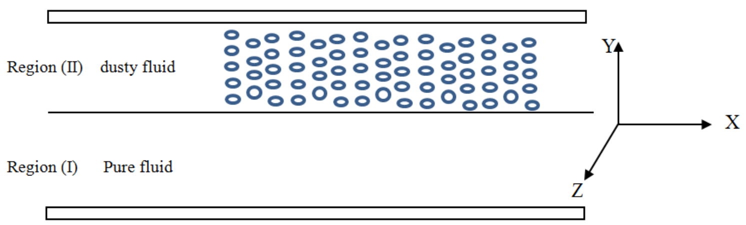


Fig. 1: Physical model.

The temperature of the lower and upper plate are considered to be constants but different and equal T_{pl1}, T_{pl2} respectively and the flow of fluids is under the influence of pressure gradient dp/dx which is applied at t=0 and its value is constant. A uniform suction from above and a uniform injection from below are applied in turn the y component of the velocity of the fluids.

The suspended particles in region two are assumed to be electrically non-conducting, spherical in shape, same radius and mass, un-deformable, and uniformly distributed throughout the flow. The particle phase is considered to be relatively diluted so that the particle-particle interaction is neglected. The concentration of particles is very small so that the net effect of the dust on the fluid particles is equivalent to an extra force $KN(u_2 - u_p)$ per unit volume. Where K is the stoke's drag constant ($k = 6\pi\mu r a$) for spherical particles of radius a, μ is the coefficient of fluid viscosity and N is the density number of particles per unit volume of the fluid.

Under these assumptions, the governing equations of motion and energy for the two fluids and particles will be mentioned in the following section.

The Governing equations:

Region-I:

The continuity equation of the fluid is written as;

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \quad (1)$$

Since the plates are infinite in the x and z directions, the physical quantities do not change in these directions which lead to one-dimensional problem and the equation reduced to

$$\frac{\partial v_1}{\partial y} = 0 \quad (2)$$

The momentum equation of the fluid in region (I) is

$$\rho_1\left(\frac{\partial u_1}{\partial t} + v_{0\,1}\frac{\partial u_1}{\partial y}\right) = -\frac{\partial p_1}{\partial x} + \mu_1\frac{\partial^2 u_1}{\partial y^2} \quad (3)$$

The energy equation of the fluid in region (I) is

$$\rho_1 C_{p1}\left(\frac{\partial T_1}{\partial t} + v_{o1}\frac{\partial T_1}{\partial y}\right) = k_1\frac{\partial^2 T_1}{\partial y^2} + \mu_1\left(\frac{\partial u_1}{\partial y}\right)^2 \quad (4)$$

Region-II:

The continuity equation of the fluid is written as;

$$\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial w_2}{\partial z} = 0 \quad (5)$$

Also, due to our assumptions the equation is reduced to

$$\frac{\partial v_2}{\partial y} = 0 \quad (6)$$

The momentum equation of the fluid is

$$\rho_2\left(\frac{\partial u_2}{\partial t} + v_{0\,2}\frac{\partial u_2}{\partial y}\right) = -\frac{\partial p_2}{\partial x} + \mu_2\frac{\partial^2 u_2}{\partial y^2} - KN(u_2 - u_{p2}) \quad (7)$$

The energy equation of the fluid is

$$\rho_2 C_{p2}\left(\frac{\partial T_2}{\partial t} + v_{o2}\frac{\partial T_2}{\partial y}\right) = k_2\frac{\partial^2 T_2}{\partial y} + \mu_2\left(\frac{\partial u_2}{\partial y}\right)^2 + \frac{\rho_p C_S}{\gamma T}\left(T_{p2} - T_2\right) \quad (8)$$

Applying Newton's second law in the *x* direction gives the equation of motion of the dust particles in the form

$$m_p\frac{\partial u_p}{\partial t} = k(u_2 - u_p) \quad (9)$$

Where: m_p is the average mass of dust particles.

And the energy equation of the particles is written as

$$\frac{\partial T_p}{\partial t} = \frac{-1}{\gamma T}(T_p - T_2) \quad (10)$$

Where T_2 is the temperature of the fluid in region (II),

T_p is the temperature of the dust particles,

ρ_p is the mass of dust particles per unit volume of the fluid,

γ_T is the temperature relaxation time, and
Cs is the specific heat capacity of the particles.

The temperature relaxation time depends on the geometry of the dust particles which is as mentioned before to be spherical in shape.

The last term in Eq. (8) is equal to $4\pi aNk(Tp - T2)$. Then

$$\gamma_T = \frac{3\text{Pr}\gamma c_s}{2C_p}$$

Where, γ is the velocity relaxation time which equal = $\frac{2\rho_s a^2}{9\mu}$ and ρ_s is the material density of dust particles = $\frac{3\rho_p}{4\pi a^3 N}$.

And thus, the initial, boundary and interface conditions can be mentioned as:

At $t \le 0$: $u_1 = u_2 = u_p = 0, \quad T_1 = T_2 = T_p = T_{pl1}$ (11a)

$At\ t > 0\ \ at\ y = -h\ \ u_1 = 0; \quad T_1 = T_{pl1}$ (11b)

at $y = 0\ u_1 = u_2;\ \ T_1 = T_2 = T_p,\ \mu_1 \frac{\partial u_1}{\partial y} = \mu_2 \frac{\partial u_2}{\partial y};\ \ k_1 \frac{\partial T_1}{\partial y} = k_2 \frac{\partial T_2}{\partial y};$ (11c)

$at\ y = h \quad u = u_p = U_o\ ; \quad T_2 = T_p = T_{pl2}$ (11d)

The problem is simplified by writing the equations in the non-dimensional form; we define the following non-dimensional quantities as:

$(\hat{x}, \hat{y}) = \frac{(x,y)}{h}, \quad \hat{t} = \frac{tU_o}{h}, \quad \hat{u} = \frac{u}{U_o}, \quad \hat{u}_p = \frac{u_p}{U_o}, \quad \hat{P} = \frac{P}{\mu U_o^2}, \quad \hat{T}_1 = \frac{T_1 - T_{\text{pl2}}}{T_{\text{pl1}} - T_{\text{pl2}}}, \quad \hat{T}_2 = \frac{T_2 - T_{\text{pl2}}}{T_{\text{pl1}} - T_{\text{pl2}}},$ $\hat{T}_p = \frac{T_p - T_{\text{pl2}}}{T_{\text{pl1}} - T_{\text{pl2}}}$ $S = \frac{\rho h v_o}{\mu}$ is the suction parameter, $\text{Re} = U_o \rho_1 h/\mu_1$ is the Reynolds number, $\text{Ec} = U_o^2/C\text{p}(T_{\text{pl2}} - T_{\text{pl1}})$ is the Eckert number, $G = m_p U_o/\text{h}K$ is the particle mass parameter, $R = KNh^2/\mu$ is the particle concentration parameter, $L_o = \text{h}/\gamma_T U_o$ is the temperature relaxation time parameter, $Pr = \frac{C_{p_1}\mu_1}{k_1}$ is the prntdle number, $\rho_R = \rho_2/\rho_1$ is the ratio of density of the two fluids, $\mu_R = \mu_2/\mu_1$ is the ratio of viscosity of the two fluids, $k_R = k_2/k_1$ is the ratio of thermal conductivity of the two fluids, $Cp_R = Cp_2/Cp_1$ is the ratio of specific heat at constant pressure of the two fluids. Using the non-dimensional parameters, the equations are rewritten as (the hats are dropped for simplicity) :

Region-I:

$$\left(\frac{\partial u_1}{\partial t} + S_1 \frac{\partial u_1}{\partial y}\right) = -\frac{\partial p_1}{\partial x} + \frac{1}{Re}\frac{\partial^2 u_1}{\partial y^2} - \left(\frac{R}{Re}\right)u_1 \quad (12)$$

$$\left(\frac{\partial T_1}{\partial t} + S_1 \frac{\partial T_1}{\partial y}\right) = \frac{1}{\text{Pr}\,Re}\frac{\partial^2 T_1}{\partial y^2} + \frac{Ec}{Re}\left(\frac{\partial u_1}{\partial y}\right)^2 \quad (13)$$

Region-II:

$$\left(\frac{\partial u_2}{\partial t} + S_2 \frac{\partial u_2}{\partial y}\right) = -\frac{\partial p_2}{\partial x} + \frac{\mu_R}{Re\rho_R}\frac{\partial^2 u_2}{\partial y^2} - \left(\frac{R}{Re\rho_R}\right)(u_2 - u_p) \quad (14)$$

$$\left(\frac{\partial T_2}{\partial t} + S_2 \frac{\partial T_2}{\partial y}\right) = \frac{k_R}{Pr\,Re\rho_R Cp_R}\frac{\partial^2 T_2}{\partial y^2} + \frac{Ec\mu_R}{Re\rho_R Cp_R}\left(\frac{\partial u_1}{\partial y}\right)^2 + \frac{2Rk_R}{3PrRe\rho_R\mu_R Cp_R}(T_{p2} - T_2) \quad (15)$$

The momentum equation of the particles is

$$\frac{\partial u_p}{\partial t} = \frac{1}{G}(u_2 - u_p) \quad (16)$$

And the energy equation of the particles is

$$\frac{\partial T_p}{\partial t} = -L_o(T_p - T_2) \quad (17)$$

And the initial, boundary and interface conditions can be rewritten as:

At $t \le 0$: $u_1 = u_2 = u_p = 0, \quad T_1 = T_2 = T_p = 0$ (18a)

$At\ t > 0\ \ at\ y = -h\ \ u_1 = 0; \quad T_1 = 0$ (18b)

at $y = 0\ u_1 = u_2;\ \ T_1 = T_2 = T_p,\ \mu_1 \frac{\partial u_1}{\partial y} = \mu_2 \frac{\partial u_2}{\partial y};\ \ k_1 \frac{\partial T_1}{\partial y} = k_2 \frac{\partial T_2}{\partial y};$ (18c)

$at\ y = h \quad u = u_p = 1\ ; \quad T_2 = T_p = 1$ (18d)

umerical Solution:

Equations (12) -(17) present a coupled non – linear system of partial differential equations and are to be solved by using initial, interface and boundary conditions (18). The exact solutions are so difficult. So, the equations will be solved using the Crank Nicholson method. linearization technique will be applied to remove the non–linear terms at a linear stage, subsequent iterative steps are made until the convergence is reached. Then the Crank Nicholson implicit method is used at two successive time levels.

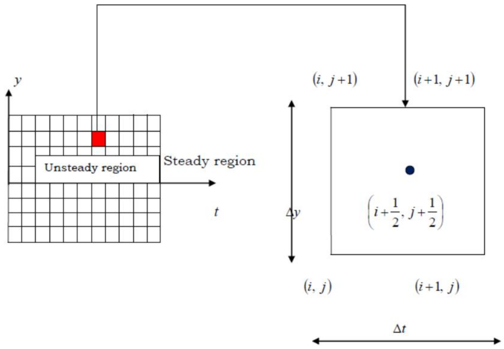


Fig. 2: concept of Crank Nicholson method.

An iterative scheme is used to solve the linearized system of difference equations. The solution at a certain time step is chosen as an initial guess for next time step and the iterations are continued till convergence, within a suitable accuracy. Finally, the resulting block tri-diagonal system is solved using the generalized Thomas – algorithm [19].

Finite difference equations relating the variables are obtained by writing the equations at the midpoint of the computational cell and then replacing the different terms by their second order central difference approximations in the *y* direction. The diffusion terms are replaced by the average of the central differences at two successive times – levels. The computational domain is divided into meshes of dimension Δt and Δy in time and space respectively as shown in figure (2). The finite difference representations for the resulting differential equations take the following forms.

- **Region One:**

$$\frac{u_{1i}^{n+1}-u_{1i}^{n}}{\Delta t}+S_1\left(\frac{u_{1i+1}^{n+1}-u_{1i-1}^{n+1}+u_{1i+1}^{n}-u_{1i-1}^{n}}{4\Delta y}\right)=Pc1+\frac{1}{Re}\left(\frac{u_{1i+1}^{n}-2u_{1i}^{n}+u_{1i-1}^{n}}{2\Delta y^2}+\frac{u_{1i+1}^{n+1}-2u_{1i}^{n+1}+u_{1i-1}^{n+1}}{2\Delta y^2}\right) \quad (19)$$

$$\frac{T_{1i}^{n+1}-T_{1i}^{n}}{\Delta t}+S_2\left(\frac{T_{1i+1}^{n+1}-T_{1i-1}^{n+1}+T_{1i+1}^{n}-T_{1i-1}^{n}}{4\Delta y}\right)\frac{1}{Re\,Pr}\left(\frac{T_{1i+1}^{n+1}-2T_{1i}^{n+1}+T_{1i-1}^{n+1}}{2\Delta y^2}+\frac{T_{1i+1}^{n}-2T_{1i}^{n}+T_{1i-1}^{n}}{2\Delta y^2}\right)+$$
$$\frac{Ec}{Re}\left(\frac{u_{1i+1}^{n+1}-u_{1i-1}^{n+1}+u_{1i+1}^{n}-u_{1i-1}^{n}}{4\Delta y}\right)^2 \quad (20)$$

- **Region Two:**

$$\frac{u_{2i}^{n+1}-u_{2i}^{n}}{\Delta t}+S\left(\frac{u_{2i+1}^{n+1}-u_{2i-1}^{n+1}+u_{2i+1}^{n}-u_{2i-1}^{n}}{4\Delta y}\right)=Pc2+\frac{\mu_R}{R_e\rho_R}\left(\frac{u_{2i+1}^{n}-2u_{2i}^{n}+u_{2i-1}^{n}}{2\Delta y^2}+\frac{u_{2i+1}^{n+1}-2u_{2i}^{n+1}+u_{2i-1}^{n+1}}{2\Delta y^2}\right)-$$
$$\left(\frac{R}{R_e\rho_R}\right)\left[\left(\frac{u_{2i}^{n+1}+u_{2i}^{n}}{2}\right)-\left(\frac{u_{2pi}^{n+1}+u_{2pi}^{n}}{2}\right)\right] \quad (21)$$

$$\frac{T_{2i}^{n+1}-T_{2i}^{n}}{\Delta t}+S\left(\frac{T_{2i+1}^{n+1}-T_{2i-1}^{n+1}+T_{2i+1}^{n}-T_{2i-1}^{n}}{4\Delta y}\right)=\frac{k_R}{Re\,Pr\rho_R Cp_R}\left(\frac{T_{2i+1}^{n+1}-2T_{2i}^{n+1}+T_{2i-1}^{n+1}}{2\Delta y^2}+\frac{T_{2i+1}^{n}-2T_{2i}^{n}+T_{2i-1}^{n}}{2\Delta y^2}\right)+$$
$$\frac{Ec\mu_R}{Re\rho_R Cp_R}\left(\frac{u_{2i+1}^{n+1}-u_{2i-1}^{n+1}+u_{2i+1}^{n}-u_{2i-1}^{n}}{4\Delta y}\right)^2+\frac{2Rk_R}{3PrR_e\rho_R\mu_R Cp_R}\left[\left(\frac{T_i^{n+1}+T_i^{n}}{2}\right)-\left(\frac{T_{pi}^{n+1}+T_{pi}^{n}}{2}\right)\right] \quad (22)$$

$$\frac{u_{2i}^{n+1}-u_{2i}^{n}}{\Delta t}=\frac{1}{G}\left[\left(\frac{u_{2i}^{n+1}+u_{2i}^{n}}{2}\right)-\left(\frac{u_{pi}^{n+1}+u_{pi}^{n}}{2}\right)\right] \quad (23)$$

$$\frac{T_{pi}^{n+1}-T_{pi}^{n}}{\Delta t}=-L_o\left[\left(\frac{T_{pi}^{n+1}+T_{pi}^{n}}{2}\right)-\left(\frac{T_{2i}^{n+1}+T_{2i}^{n}}{2}\right)\right] \quad (24)$$

Results and Discussion

In this section, the results of unsteady Couette flow of two viscous, incompressible, immiscible fluids between two infinite parallel horizontal plates with suspended particles are presented and analyzed for different parameters. In order to obtain a good idea about the behavior of fluids and particles, both of velocity and temperature is graphically represented.

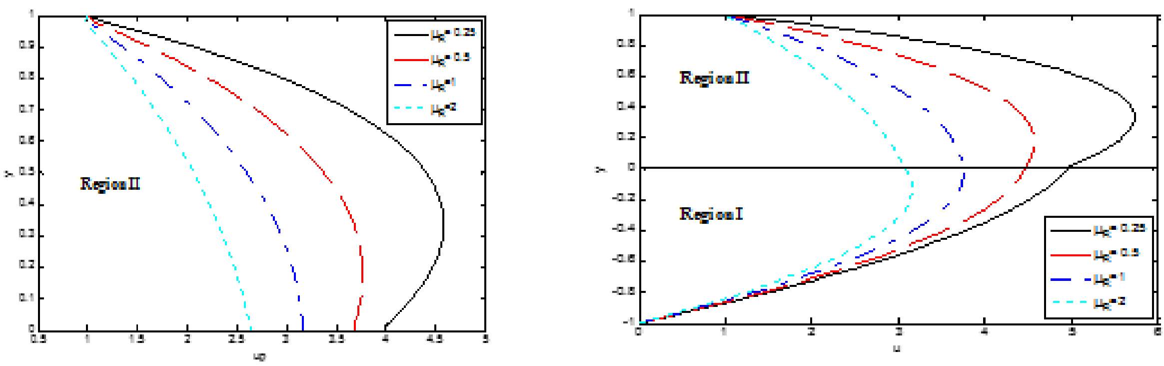
Figure (3), illustrates the effect of viscosity ratio of fluids μ_R on the velocity of the fluid and the suspended particles the particles in the two regions within the following parameters $S_1 = S_2 = 1$, Ec = 0.2; $R = 1$; $Pr = 1$; $\rho_R = 1$; $k_R = 1$; $Cp_R = 1$. Figure 3 (a) shows that, the velocity of the fluid u is increasing within decreasing of the viscosity ratio of the two fluids. It can be noticed that while the viscosity ratio of the two fluids is decreasing, the velocity of the suspended particles is increasing as shown in fig.(3b).

Figure (4) presents the effect of viscosity ratio on the temperature of the fluid T in the two regions. It is observed that, both of the temperature of the fluid is increasing within decreasing of the viscosity ratio of the two fluids. This is due to the fact that as the viscous ratio increase, the fluids in both regions become thicker and hence, the flow velocity though the plats is reduced causing the temperature distribution to decrease.

The velocity profiles and temperature distribution of the fluid for different the values of density ratio ρ_R are shown in figures (5) and (6). It is observed that the velocity of the fluid is increasing within increasing of the density ratio of the two fluids. Figure (6) presents the effect of density ratio on the fluid temperature. It can be seen that, fluid temperature is increasing as the density ratio of the fluid decrease. The influence of thermal conductivity ratio k_R of fluids on the fluid temperature is presented in figure (7). The temperature of the fluid in the two regions is increasing as thermal conductivity ratio is decreasing, this thermal suppression is large in region-I compared to region-II because the temperature boundary conditions are different.

Figures (8) and (9) indicate the effect of specific heat of fluids Cp_R and Prantdl number Pr on the fluid temperatures. It can be noticed that, the fluid temperature is increasing as the specific heat of the fluid decrease. Also, it is observed that, the fluid temperature decrease with decreasing Prantdl number.

Figure (10) presents the influence of Eckert number Ec on the fluid temperatures. It can be declared that both of the fluid and particles temperatures are increasing as the Eckert number decrease. This is an expected behavior as seen from the energy equations.



(a) (b)

Fig. 3: Effect of viscosity ratio on Fluid and particle velocity.
(a)Fluid velocity; (b) Particle velocity.

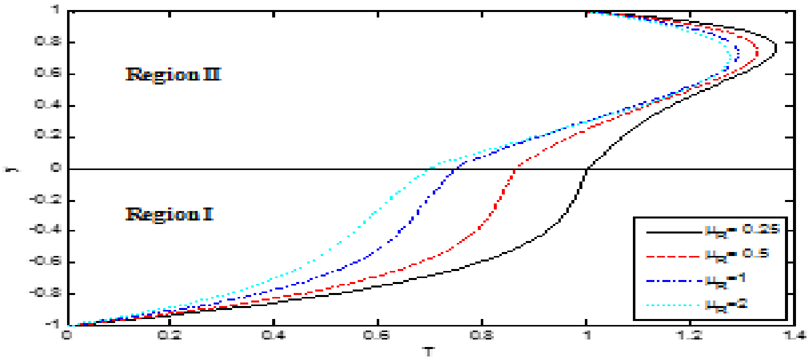


Fig. 4: Effect of viscosity ratio on fluid temperature.

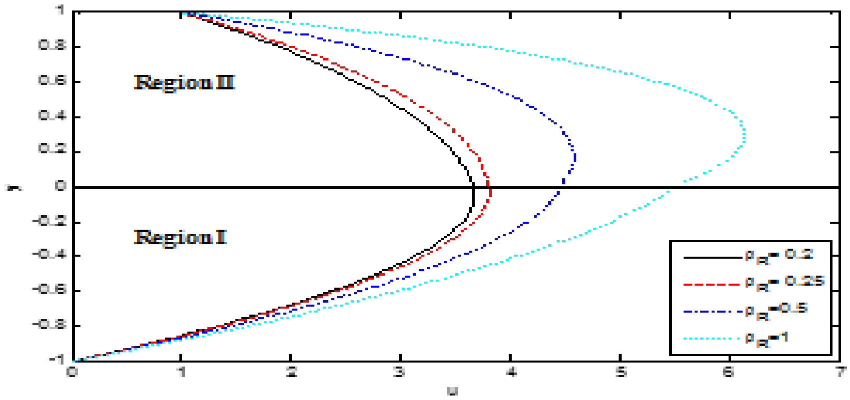


Fig. 5: Effect of density ratio on the fluid velocity.

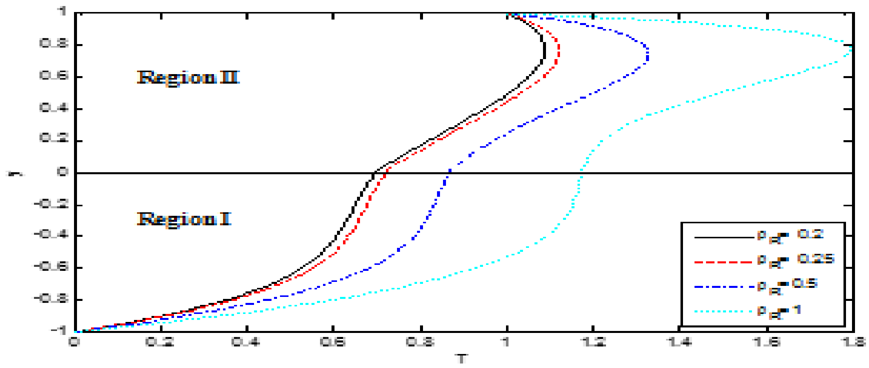


Fig. 6: Effect of density ratio on the fluid temperature.

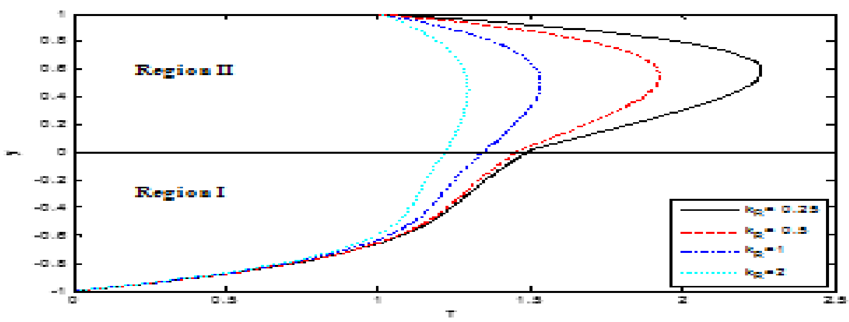


Fig. 7: Effect of thermal conductivity ratio on the fluid temperature.

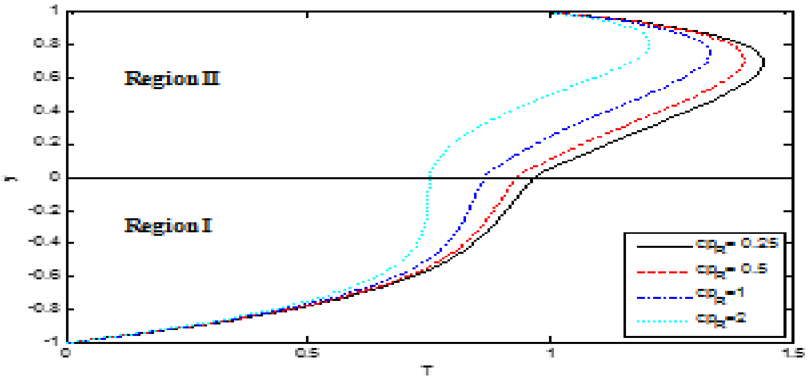


Fig. 8: Effect of specific heat ratio on the fluid temperature.

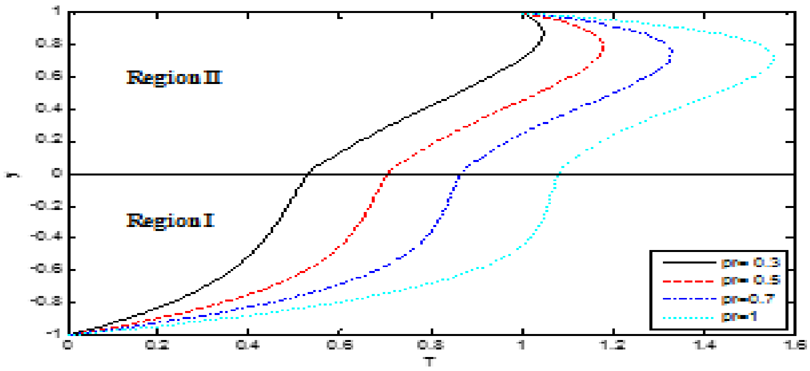


Fig. 9: Effect of Prndtle number on the fluid temperature.

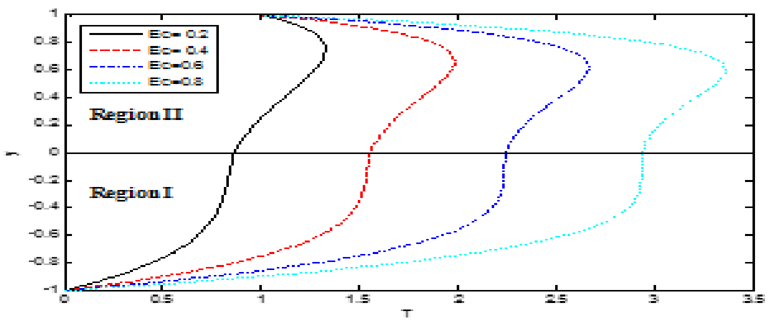


Fig. 10: Illustrates the effect of Eckert number on the fluid temperature.

Conclusion

The couette flow of two viscous, incompressible and immiscible, electrically conducting, dusty fluids between two infinite parallel horizontal plates is investigated. The fluids are considered to flow under the effect of pressure gradient and temperature gradient. The partial differential equations governing the flow and heat transfer are solved numerically using finite difference method with crank Nicolson technique for both regions of fluids. Effects of physical parameters such as viscosity ratio, conductivity ratio, density ratio, Eckert number and Prandtl number on the velocity and temperature fields for the two fluids and the particles are obtained and represented graphically and it can be concluded that:

- The effect of viscosity ratio μ_R on the velocity and temperature of the fluid and particles has been studied and it is found that as the viscosity ratio increase, both velocity and temperature decrease.

- The velocity and temperature of the fluids increase with increasing density ratio.
- The temperature of the fluid increase with decreasing of both thermal conductivity k_R and specific heat ratio Cp_R .
- Both of Eckert number Ec and Prandtl number Pr tends to enhance fluids and particles temperature.

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