

Modeling of Proton-Induced Fission Cross Section of Some Target Nuclei

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ABSTRACT

Mathematical modeling for proton-induced fission cross section of some target nuclei at different bombarding proton energies ranging from ~ 10 to ~ 10000 MeV has been investigated. The fission cross section has calculated through the novel application of a cascade correlation neural network (CCNN) model based on the back-propagation method. The fission cross section is calculated as a function of the mass number, atomic number and the incident proton energy. Promising results were obtained in terms of accuracy using the Neural Network model. It is observed that the (CCNN) model performed well in describing the fission cross section.

Key words: Proton-Induced Fission Cross Section, Mathematical Modeling, Cascade Correlation Neural Network

Introduction

Since the discovery of nuclear fission from more than seven decades, considerable experimental and theoretical works have been undertaken on this process. There is a high neutron multiplicities observed in fission experiments explained (Hinde *et al.*, 1992) and a dynamical description of the fission process seems more appropriate (Fröbrich *et al.* 1993). This dynamical process was already proposed (Kramers 1940), who introduced into fission process the concept of dissipation energy. The dissipative effect were observed from pre- and post-scission neutron gamma ray (Hofman 1994), neutron multiplicities, (Hilsher and Rossner 1992) and charged particles, (Lestone 1993). Recently, it has also been established that dissipative effects can explain fusion-fission cross sections at low excitation energies, (Lestone and McCalla 2009). Unfortunately Jurado *et al.* (2005) fails to use the fission picture (Kramers 1940) at high excitation energies, where nuclei need a certain time (transient time or delay time) to explore the potential-energy landscape. Therefore, the stationary fission width is suppressed at shorter times (Grange, Jun-Qing and Weidenmüller 1983) requiring a certain transient time to establish a quasi-stationary flux over the fission barrier. The transient time effects was investigated within fission cross sections of spallation reaction (Jurado *et al.* 2003, Benlliure *et al.* (2006) and Ayyad *et al.* 2014) which gives evidences of transient effects at high excitation energies of proton and deuterium induced fission from actinides to subactinides. However, the experimental setup (Boutoux *et al.* 2013, Jurado *et al.* 2004, Schmitt *et al.* 2010 and Rodriguez- Sánchez *et al.* 2015) gives a new observations to shed light on dynamical effects of fission and fission fragments. Nowadays, the fission cross section plays an important role in nuclear reaction physics. So, in this context a new mathematical model introduced to calculate and predict the fission cross section due to the induced proton through some target nuclei in terms of their mass number, atomic number and the energy of the incident proton.

Neural Networks are capable of capturing the underlying variation in the data and making effective use of them for classification and prediction. Neural Networks have found its applications in various fields of science and are effective machine learning tool. The advantage of Neural Networks is that it is a powerful data-modeling tool, which has the ability to recognize patterns even if there is no functional relationship between input and output. We have tried to explore the possibility of the use of neural networks for fission cross section prediction.

This paper discusses the possibility of employing Neural Networks for prediction of proton-induced fission cross section from experimental data. The CCNN has been employed for prediction of proton-induced fission cross section.

The paper is organized as follows: In the next section the modelling method of fission cross section is described. The results obtained are presented in third Section. Finally, the findings and conclusions in the last section.

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Fission cross section model building based on cascade-correlation neural networks

An artificial neural network is a computing system that consists of a collection of artificial neurons connected with each other. An artificial neuron simulates performance of a biological neuron. Such artificial neuron was first introduced in (McCulloch and Pitts, 1943). Neural networks are an alternative method for prediction and classification. The BNNs (back propagate neural network) are feed-forward network trained by propagation of error back through the networks. The network size and architecture (number of processing units, layers, and interconnections) must be determined before training. Disadvantages of BNNs include the determination of the optimal network architecture and long training times. The cascade correlation neural network (CCNN) was developed to alleviate these problems. The CCNN learning algorithm has been proposed in (Fahlman and Lebiere, 1990). This algorithm not only constructs a network but also trains the weights. With this, the number of hidden layers is not assigned in advance, but is determined during the process of learning.

The CCNN configures its own architecture as it trains. It starts with a minimal network (i.e., input neurons and output neurons). The CCNN then sequentially adds hidden units until the error decreases below a user-defined threshold. Each new hidden unit is connected to the network inputs and the outputs form the previously installed hidden units. Therefore the outputs from the previously added hidden units cascade into each new unit.

The inputs ($x = \{\text{Proton energy, } Z \text{ and } A\}$) are multiplied by the weights and adjusted by a bias,

$$O_{ij} = \text{fission cross section} = f(w, x) = f(\text{net}_{ij}) = f\left(\sum_{m=1}^v w_{ij}x_m + b_j\right) \quad (1)$$

Where v is the number of input connections to unit j , w_{jm} is a component of the normalized weight, and x_m is the input activation coming from the m^{th} neuron in the preceding layer. The results (net_{ij}) are input to the transfer function (in our case we have used the sigmoid function); the outputs of the j^{th} hidden unit (o_{ij}) is obtained by

$$O_{ij} = f(w, x) = f(\text{net}_{ij}) = \frac{1}{1 + e^{-\text{net}_{ij}}} \quad (2)$$

The CCNN training maximizes covariance between candidate neuron output and its residual error. The weights for hidden units are trained by maximizing the covariance between the unit's output and the residue error. The covariance magnitude ($|C_j|$) of the output from candidate unit j and the residual error from output k is obtained from

$$|C_j| = \sum_{k=1}^p \sum_{i=1}^n (O_{ij} - \bar{O}_j) (e_{ik} - \bar{e}_k) \quad (3)$$

For which the covariance is calculated with respect to the n observations in the training set. The absolute values of the covariances are added for the p output units. The averages are obtained for the n objects in the training set for the hidden unit output (o_j) and error (e_j). The denominator of $n - 1$ is omitted from the calculation, because it is constant through the entire training procedure. The weights are adjusted so that $|C_j|$ is maximized.

Several candidate units can be trained in parallel, and the one with largest covariance can be selected as the next hidden unit to install into the network. Once trained, the hidden units no longer are adjusted; thus only one unit is trained at a time. Statistical metrics such as, correlation coefficient (R^2) and the normalized mean square error (NMSE), were used to evaluate the prediction performance of the CCNN model. RMS and R^2 were used to measure the deviation between the actual and calculated values.

The process of network learning consists of six steps. They are listed below:

- Step 1.** Train the initial net until the mean square error E reaches a minimum.
- Step 2.** Install a hidden candidate node. Initialise weights and learning constants.
- Step 3.** Train the hidden candidate node. Stop if the correlation between its output and the network output error is maximized.
- Step 4.** Add the hidden candidate unit to the main net, i.e. freeze its weights, connect it to the other hidden units, if available, and connect to the network outputs.
- Step 5.** Train the main net that includes a hidden unit. Stop if the minimum mean square error is reached.
- Step 6.** Add another hidden unit. Do Steps 2-5 until the mean square error value is acceptable for solving the given task.

Results and Discussion

In the deformed state of a nucleus there are two forces acting on the nucleus. The surface energy and the coulomb repulsion between the fission fragments. These two forces produce a potential barrier. In case of neutron or proton bombard a nucleus. The target nucleus absorbs the neutron or proton it will bend to the target and releasing energy (binding energy of the proton or neutron) in the form of vibrational energy which could be more than the potential barrier energy in order to overcome this barrier. When the binding energy of this neutron or proton is sufficient to overcome this barrier energy, the incident neutron or proton needs to have a minimum kinetic energy in order to be able to induce fission.

In the present work I am going to use Cascade Correlation Neural Network Model to calculate the fission cross section as a function of the mass number (A), atomic number (Z) and the energy of the incident proton. The CCNN calculations are compared to the experimental data (Prokofiev 2001) and to the calculations of Mircea I. Baznat and Konstantin K. Gudima (2003). The energy of the incident proton should be high enough to produce fission (here it is more than 50 MeV) to overcome the potential barrier and producing fission. The CCNN calculations for the fission cross section is carried out on the preactinide nuclei ^{181}Ta , ^{195}Pt , ^{208}Pb and ^{209}Bi as shown in fig. 1(a,b,c,d). In fig. 1(a,b) the first two nuclei ^{181}Ta and ^{195}Pt , the experimental data of the proton incident energy are ranged from ~ 200 MeV to ~ 6000 MeV. The fission cross section is obtained in CCNN theoretical calculations (CCNN Model-Our model) which shows an excellent agreement with the experimental data (Experimental) as well as the calculations of Mircea I. Baznat and Konstantin K. Gudima(2003) (Theoretical Model). In the same fig. 1(c,d) the other two nuclei ^{208}Pb and ^{209}Bi also having an experimental data of the proton incident energy to obtain the fission cross section ranged from ~ 50 MeV to ~ 10000 MeV. The CCNN calculations are in excellent agreement with the calculations of Mircea I. Baznat and Konstantin K. Gudima (2003) and also with the experimental data. It is also clear in fig. 1(a,b,c,d), that CCNN results for the fission cross section are affected by the change in the mass number and the atomic number which have a smooth trend with different slopes in the curves as we see in this figure.

The CCNN was trained for the experimental data (Prokofiev, A. V. 2001), for different values of the goal parameter. The best results were achieved when the network was trained using the following network training parameters are used: The type of activation functions used in the hidden and output layers are the Gaussian and the linear functions, respectively, minimum neurons = 1, maximum neurons = 8, candidate Neurons = 8, maximum steps without improvement = 10, and Over-fitting protection control = 10-fold cross-validation.

Candidate neurons are the list of neuron from which hidden units are successively chosen and added to the network. minimum number of neurons show how many hidden neurons must be added to the network. maximum number of neurons shows the maximum number of hidden neurons that can be added to the network. It is same as the number of candidate neurons, we can add one or more (even all) the candidate neurons to the network. The training process is continued till the results don't improve for a certain number of times, denoted by the maximum steps without improvement parameter. The CCNN have this disadvantage of over-fitting the training data. Due to this, the accuracy values are quite high in case of training data, but low in testing data. So, for preventing this, the model is validated as it grows (10-fold cross validation is used). As stated earlier, the values which yield the best results are taken for testing As started earlier, the values which yeild the best results are taken for testing. The prediction Accuracy achieved was correlation coefficient (R^2) and the normalized mean square error (NMSE), values reported in the training stage are 0.99 and 0.001246, respectively. The R^2 and the normalized mean square error (NMSE) values reported in the validation stage are 0.97 and 0.055, respectively.

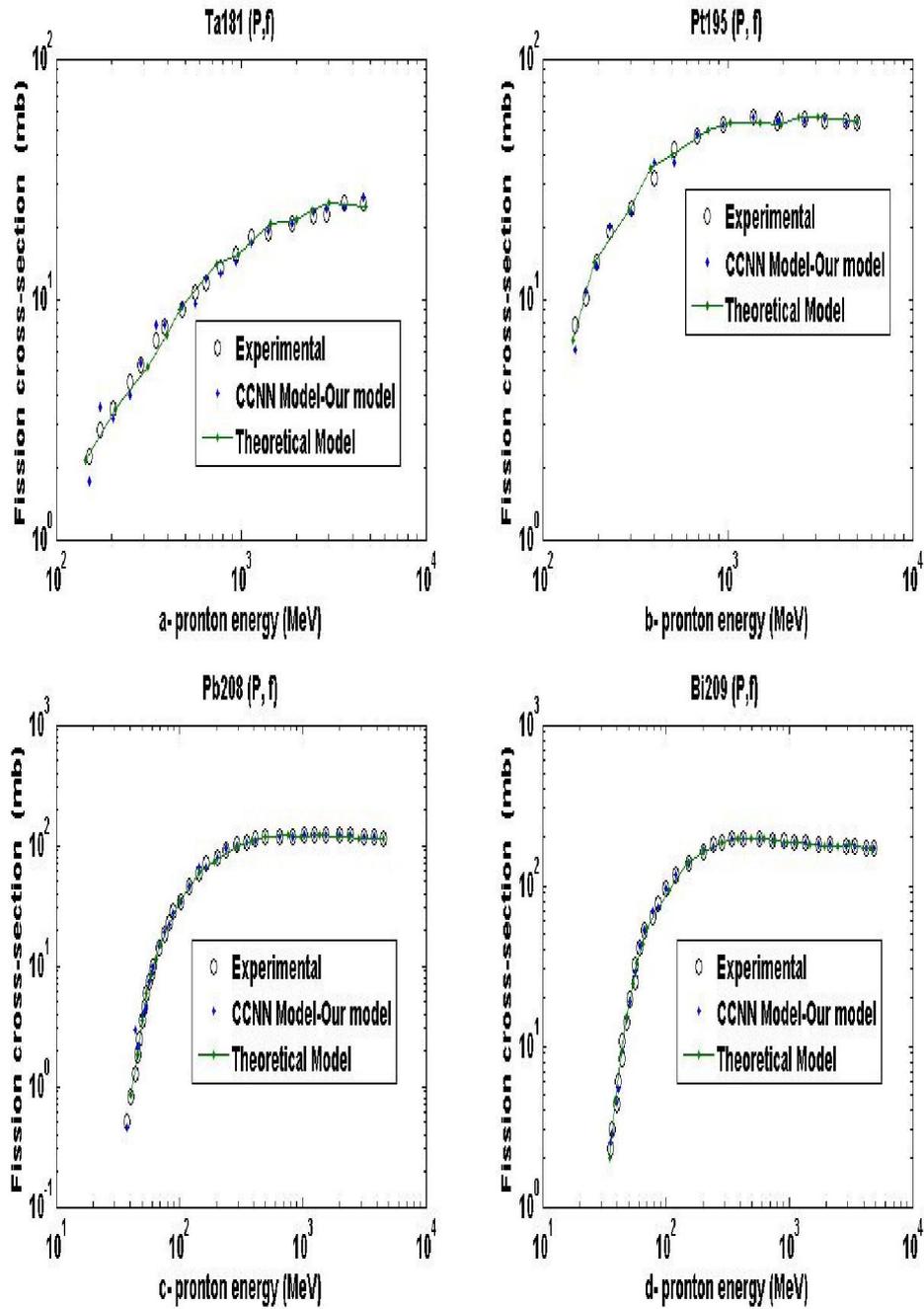


Fig. 1(a,b,c,d):Represent a comparison of our model of calculations (CCNN Model-Our model)) and the calculations of Mircea I. Baznat and Konstantin K. Gudima (2003) (Theoretical Model) with the experimental data (Experimental) for the preactinide nuclei (Prokofiev, A. V. 2001) ^{181}Ta , ^{195}Pt , ^{208}Pb and ^{209}Bi .

Conclusion

The description for proton-induced fission cross sections in the present energy region has been investigated using a new technique called cascade correlation neural network. Present calculations can reproduce an excellent agreement with the experimental data of fission cross section and other theoretical calculations. At this moment, the accuracy of present results obtained using our model encourage us to make more calculations for the fission cross section in other regions of different mass numbers. Further investigation is necessary by comparing with other theoretical and/or semi-empirical approaches. To utilize present results, a trial study is in progress.

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