Current Science International Volume : 06 | Issue : 04| Oct.- Dec. 2017
ISSN: 2077-4435 Pages: 847-856

Nonlinear Analysis of Quasi Static Consolidation for Fully Saturated Porous Media Subjected to Dynamic Loads

Ramadan Awadalla[1], W. Abbas[2], S. Bicher[3], Mostafa A.M. Abdeem[3] and E.S.M. El Shinnawy[1]

[1]*Faculty of Engineering (Mataria), Engineering Physics and Mathematics Department, Helwan University, Cairo, Egypt.*
[2]*Basic and Applied Science Department, College of Engineering and Technology, Arab Academy for Science, Technology, and Maritime Transport, Cairo, Egypt.*
[3]*Faculty of Engineering, Department of Engineering Mathematics and Physics, Cairo University, Giza, Egypt.*

Received: 20 Sept. 2017 / Accepted: 28 Nov. 2017 / Publication date: 14 Dec. 2017

ABSTRACT

The behavior of geomaterials spatially in soils is governed by a set of partial differential equations caused by the interaction between solid skeleton and pore fluid. The present work, introduces the semi analytical solution of quasi static consolidation for fully saturated porous media subjected to dynamic loads. The governing equations with the corresponding boundary conditions solved analytically using the regular perturbation technique. The nonlinearity of material and geometry are considered. The mathematical formulation of the problem for the quasi-static consolidationis introduced and the parametric study is used to investigate the effect of nonlinearity of material and geometry on the pore pressure, and total stress.

Kay words: Porous media, Perturbation technique, Pore fluid, Total stress, Quasi static.

Introduction

Most of geotechnical engineering problems are focused on transient phenomena occurring in earthquakes, wave loading, and consolidation. All of these cases, the interaction between the deformation of solid skeleton of soil and pore fluid is of major importance. Also, the same behavior occurred in other area such as, biomechanics, the solid skeleton referred to porous bone structure and the pore fluid to the circulating bloods. The formulation of quasi static phenomena which describes the interaction between the solid and fluid phases was first established by Biot (1941), and he extended these formulations to include the dynamic response Biot (1956, 1962). At later date Truesdell (1957, 1960) introduced the mixture theory. For quasi static consolidation analysis, the assumption of drained and undrained behavior may be depending on the loading rapidity and permeability of soil. Zienkiewicz and Bettess (1980) presented linear quasi static formulation of two phase medium, based on the work of Boit. These formulation were applied to a simple problem of homogeneous soil layer under periodic loads and from the exact solution, the limits of various assumptions were good. After that, Zienkiewiczand Shiomi(1984)improved his formulation by inserting the nonlinear term of acceleration of pore fluid. The generalized two-phase behavior of soil was established by Zienkiewicz *et al*. (1990). The volumetric expansion of the solid-fluid phase was included.After that, Xikuiand Zienkiewicz(1992) presented the deformation of multiphase flow of porous media;he took the saturation of wetting fluid as a primary unknown of model. A fully coupled dynamic model for the analysis of water and air flow of two phases in the deformable porous media was introduced by Bernhard *et al*. (2001). Elgamal *et al*. (2002, 2003) introduced the development of the computational model for analysis of cyclic mobility; he took the shear deformation of cohesion-less soils into consideration. The two dimensional of dynamic consolidation for fully saturated soil was developmented by Hassanenand El-Hamalawi (2007). The large deformation of coupled dynamic and contact analysis of fully saturated porous media presented by Yonggang *et al*. (2013). Nedjar

Corresponding Author: Ramadan Awadalla, Faculty of Engineering (Mataria), Engineering Physics and Mathematics Department, Helwan University, Cairo, Egypt.
E-mail: Eng_RAM2010@yahoo.com

(2013), presented the formulation of a nonlinear porosity for a fully saturated porous media with finite deformation and high pore pressure conditions. Heider *et al*. (2014) introduced the dynamic response of fluid of fully saturated porous media and the application of seismic produced the soil liquefaction. Rohanand Lukes (2015) introduced the physical nonlinear model of Boit continuum, and describe the sensitivity analysis of the homogenous material coefficients with respect to microstructure deformation.Rozaand Behzad (2016) used themass conservative method for the numerical modeling of subsidence in saturated porous media. Wang *et al*. (2017) presented a semi analytical solution to one dimensional consolidation equation of fractional derivative viscoelastic saturated porous media subjected to time dependent loads. Xu *et al*. (2017) introduced the dynamic fluid and solid phases coupled problems for the saturated porous media in *u-p* equations with the fluid compressibility materials in spatial domain.

This paper introduces the nonlinear semi analytical solution of the constitutive nonlinear differential equations of the quasi static consolidation for fully saturated porous media subjected to dynamic loads. The geometry and material are assumed to be nonlinear to show the power of this method to find the closed form solution. The regular perturbation technique used to solve the governing equations with the corresponding boundary conditions. The results are discussed in detail using graphs.

Formulation of The problem:

The saturated porous media containing thesolid grains and a viscous fluid, these two materials are calledtwo phase porous media.Biot (1941, 1960)introducedthe constitutive relations governing the behavior of saturated porous media with dynamic moving loads. However, Zienkiewiczand Shiomi(1984) enhanced these relations by adding the nonlinear part of material behaviors. Therefore, he introduced the total momentum equilibrium equation for the solid-fluid assembly as:

$$\sigma_{ij,j} + \rho\, g_i = \rho\, \ddot{u}_i + \rho_f\left(\ddot{w}_i + \dot{w}_k \dot{w}_{i,k}\right) \tag{1}$$

Where, σ_{ij} is the total stress, u is the soil grains displacement, w is the pore fluid displacement, ρ is the assembly density, ρ_f is the fluid phase density, g_i is the body force acceleration.

The equation of motion of pore pressure as:

$$-p_{,i} - R_i + \rho_f b_i = \rho_f\left(\ddot{u}_i + \left(\frac{\partial \dot{w}_i}{\partial t} + \dot{w}_k \dot{w}_{i,k}\right)/n\right) \tag{2}$$

Where,p is the pore pressure,n is the porosity, and R_i represents the viscous drag force which, assuming the validity of the Darcy seepage lawis given byAttia *et al*.(2014, 2015,2016):

$$k_{ij} R_j = w_i \tag{3}$$

Where, k_{ij} define the generally anisotropic permeability coefficients. For isotropy these are conveniently changed by a single k value.

We ought to note that, the permeability may be defined as a function of strain and of external temperature as: $k_{ij} = k_{ij}\left(\left(\varepsilon_{ij} - \varepsilon^{o}{}_{ij}\right),\ T\right)$

The constitutive relations for solid skeletonare given as:

$$\dot{\sigma}_{ij} = \left(D_{ijkl}\left(\dot{\varepsilon}_{kl} - \dot{\varepsilon}^{\circ}_{ij}\right) + \dot{\omega}_{ik}\sigma_{kj} + \dot{\omega}_{jk}\sigma_{ki}\right) - \alpha\, \delta_{ij}\dot{p} \tag{4}$$

$Where\dot{\varepsilon}_{ij} = \left(\dot{u}_{i,j} + \dot{u}_{j,i}\right)/2 \quad , \quad \dot{\omega}_{ij} = \left(\dot{u}_{j,i} - \dot{u}_{i,j}\right)/2 \quad ,\& \quad \alpha \cong (1 - K_T/) \ \leq 1$

Where,$\dot{\varepsilon}_{ij}$ is the rate of deformation tensor, $\dot{\omega}_{ij}$ is the spin tensor, $\dot{\varepsilon}^{\circ}_{ij}$ is the rate of thermal of other initial strain, D_{ijkl} is the tangent modulus matrix, and K_T is the bulk modulus of porous media.

The continuity equation of flow as:

$$\dot{p}/Q + \alpha\, \dot{\varepsilon}_{ii} + \dot{w}_{i,i} + (\dot{\rho}_f/\rho) w_i + \dot{s}_0 = 0 \tag{5}$$

Where,$\dot{s}_0$ represents the rate of volume change of fluid such as may be caused by thermal change, etc., the term Q represents the combined compressibility of fluid and solid phases, which can be defined by the relation containing the bulk modulus of each component as introduced by Biot and Willis (1957), Zienkiewicz (1982).

$$1/Q = n/K_S + (\alpha - n)/\, K_f \tag{6}$$

Where, K_S is thebulk modulus of solid phase,K_f is the bulk modulus of the fluid.

Figure 1 represents a formulation of fully saturated porous media subjected to dynamic loads, in this work,the set of partial differential equations for quasi static of consolidation will be formulated in the vertical direction only, and the nonlinearity for material and geometry will be considered.In this case the motion is slow phenomena; therefore, all acceleration terms for solid-fluid phases will be neglected. The perturbation technique will be used in order to solve this system of nonlinear partial differential equations.

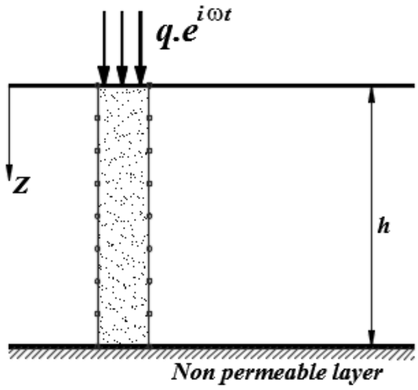

Fig. 1: Fully saturated porous mediaunderdynamic loads

The quasi static nonlinear partial differential equations become:

$$\frac{\partial \sigma_{zz}(z,t)}{\partial z} = \rho_f \frac{\partial w(z,t)}{\partial t}\frac{\partial^2 w(z,t)}{\partial z\, \partial t} \tag{7}$$

$$-\frac{\partial p(z,t)}{\partial z} = \frac{\rho_f}{n}\frac{\partial w(z,t)}{\partial t}\frac{\partial^2 w(z,t)}{\partial z\, \partial t} + \frac{\rho_f\, g}{k}\frac{\partial w(z,t)}{\partial t} \tag{8}$$

$$\frac{\partial \sigma_{zz}(z,t)}{\partial t} = D(z)\frac{\partial^2 u(z,t)}{\partial z\, \partial t} - \alpha\frac{\partial p(z,t)}{\partial t} \tag{9}$$

$$-\frac{1}{Q}\frac{\partial p(z,t)}{\partial t} = \alpha\frac{\partial^2 u(z,t)}{\partial z\, \partial t} + \frac{\partial^2 w(z,t)}{\partial z\, \partial t} \tag{10}$$

The boundary conditions of a fully saturated soilare:

At the surface (z=0):

$$p(0,t) = 0 \quad , \quad \sigma_{zz}(0,t) = q.\,e^{i\omega t} \quad , \quad \frac{\partial u(0,t)}{\partial z} = \frac{q}{D_o}.\,e^{i\omega t} \quad , and \quad \frac{\partial w(0,t)}{\partial z} = -\frac{q}{D_o}.\,e^{i\omega t}$$

At the base (z=h):

$$\frac{\partial p(h,t)}{\partial z} = 0 \quad , \quad \frac{\partial \sigma_{zz}(h,t)}{\partial z} = 0 \quad , \quad u(h,t) = 0 \quad , and \quad w(h,t) = 0$$

Where, q is the periodic load intensity, and$D_o$is the modulusof rigidity at the surface.
We introduce the following non-dimensional form ofparameters as:

$$Z=\frac{z}{h}\quad,\quad U=\frac{u}{(qh/D_o)}\quad,\quad W=\frac{w}{(qh/D_o)}\quad,\quad \sigma_{ZZ}=\frac{\sigma_{zz}}{q}\quad,\quad P=\frac{p}{q}\quad,\quad \beta=\frac{\rho_f}{\rho}\quad,\quad \gamma=\frac{\beta}{n}\ ,$$

$$T=\frac{t}{h}V_C$$

Where,$V_C$is the compression wave velocity, and given by$V_C^2=\frac{D_o+Q}{\rho}$,

Assume the nonlinear rigidity of solid skeletonD(z)take the form:

$$D(z)=D_o\left[1+\left(\frac{q}{D_o}\right).f(Z)\right]^2$$

By using the non-dimensional quantities into the set of equations (7) through (10)get:

$$(1-K_1)\frac{\partial\sigma_{ZZ}(Z,T)}{\partial Z}=\beta\left(\frac{q}{D_o}\right)\frac{\partial W(Z,T)}{\partial T}\frac{\partial^2 W(Z,T)}{\partial Z\,\partial T} \tag{11}$$

$$-(1-K_1)\frac{\partial P(Z,T)}{\partial Z}=\gamma\left(\frac{q}{D_o}\right)\frac{\partial W(Z,T)}{\partial T}\frac{\partial^2 W(Z,T)}{\partial Z\,\partial T}+\frac{1}{\pi_1\sqrt{\pi_2}}\frac{\partial W(Z,T)}{\partial T} \tag{12}$$

$$\frac{\partial\sigma_{ZZ}(Z,T)}{\partial T}=\left[1+\left(\frac{q}{D_o}\right)f(Z)\right]^2\frac{\partial^2 U(Z,T)}{\partial Z\,\partial T}-\frac{\partial P(Z,T)}{\partial T} \tag{13}$$

$$-\frac{\partial P(Z,T)}{\partial T}=\frac{K_1}{1-K_1}\left[\frac{\partial^2 U(Z,T)}{\partial Z\,\partial T}+\frac{\partial^2 W(Z,T)}{\partial Z\,\partial T}\right] \tag{14}$$

Where,$K_1=\frac{Q}{D_o+Q}$, $\pi_1=\frac{kV_C^2}{g\beta\omega h^2}$, $\pi_2=\frac{\omega^2h^2}{V_C^2}$, *and* $\varpi={\pi_2}^{1/2}$

The regular perturbation method:

It is clear that in equations (11) through (14), the nonlinear part multiplied by the parameter(q/D_o), this parameter is very sensitive and called ζ,for the cases of engineering interest e.g. a tractor on a clay field, the value of ζranges from 0.02 to 0.05. This value creates a weakly nonlinear problem. Therefore; we can use the regular perturbation method which is presented by Farlow (1982) to solve the formulated system of nonlinear partial differential equations:

$$U(Z,T)=U_0(Z,T)+\zeta.U_1(Z,T)+0\{\zeta^2.U_2(Z,T)+\zeta^3.U_3(Z,T)+\cdots\}$$
$$W(Z,T)=W_0(Z,T)+\zeta.W_1(Z,T)+0\{\zeta^2.W_2(Z,T)+\zeta^3.W_3(Z,T)+\cdots\}$$
$$P(Z,T)=P_0(Z,T)+\zeta.P_1(Z,T)+0\{\zeta^2.P_2(Z,T)+\zeta^3.P_3(Z,T)+\cdots\}$$
$$\sigma_{ZZ}(Z,T)=\sigma_{ZZ_0}(Z,T)+\zeta.\sigma_{ZZ_1}(Z,T)+0\{\zeta^2.\sigma_{ZZ_2}(Z,T)+\zeta^3.\sigma_{ZZ_3}(Z,T)+\cdots\} \tag{15}$$

From the above definition of the perturbation method, the zero perturbation equations become:

$$(1-K_1)\frac{\partial\sigma_{ZZ_0}(Z,T)}{\partial Z}=0 \tag{16}$$

$$-(1-K_1)\frac{\partial P_0(Z,T)}{\partial Z}=\frac{1}{\pi_1\sqrt{\pi_2}}\frac{\partial W_0(Z,T)}{\partial T} \tag{17}$$

$$\frac{\partial\sigma_{ZZ_0}(Z,T)}{\partial T}=\frac{\partial^2 U_0(Z,T)}{\partial Z\,\partial T}-\frac{\partial P_0(Z,T)}{\partial T} \tag{18}$$

$$-\frac{\partial P_0(Z,T)}{\partial T}=\frac{K_1}{1-K_1}\left[\frac{\partial^2 U_0(Z,T)}{\partial Z\,\partial T}+\frac{\partial^2 W_0(Z,T)}{\partial Z\,\partial T}\right] \tag{19}$$

For a periodic loading, the solutions can be separated as:

$$U_0(Z,T)=U_0(Z)e^{i\varpi T}\quad,\quad W_0(Z,T)=W_0(Z)e^{i\varpi T}$$
$$\sigma_{ZZ_0}(Z,T)=\sigma_{ZZ_0}(Z)e^{i\varpi T}\quad,\quad P_0(Z,T)=P_0(Z)e^{i\varpi T} \tag{20}$$

Therefore, equations (16) through (19) may be written as:

$$(1-K_1)\frac{d\sigma_{ZZ_0}(Z)}{dZ}e^{i\varpi T}=0 \quad (21)$$
$$-(1-K_1)\frac{dP_0(Z)}{dZ}e^{i\varpi T}=i\varpi W_0(Z)e^{i\varpi T} \quad (22)$$
$$i\varpi\sigma_{ZZ_0}(Z)e^{i\varpi T}=i\varpi\frac{dU_0(Z)}{dZ}e^{i\varpi T}-i\varpi P_0(Z)e^{i\varpi T} \quad (23)$$
$$-i\varpi P_0(Z)e^{i\varpi T}=\frac{K_1}{1-K_1}i\varpi\left[\frac{dU_0(Z)}{dZ}+\frac{dW_0(Z)}{dZ}\right]e^{i\varpi T} \quad (24)$$

The Non-dimensional boundary conditions for zero perturbation equations become:
At the surface (Z=0):

$$P_0(0)=0\ ,\ \sigma_{ZZ_0}(0)=1\ ,\ dU_0(0)/dZ=1\ ,\ dW_0(0)/dZ=-1$$

At the base (Z=1):

$$dP_0(1)/dZ=0\ ,\ d\sigma_{ZZ_0}(1)/dZ=0\ ,\ U_0(1)=0\ ,\ W_0(1)=0 \quad (25)$$

Solving the set of system differential equations (21) through (24), the corresponding coupled second order differential equations may be obtained as:
The two equations of zero perturbation will be in these forms:

$$\left[\frac{d^2}{dZ^2}\right]U_0(Z)+\left[K_1\left(\frac{d^2}{dZ^2}\right)\right]W_0(Z)=0 \quad (26)$$
$$[K_1(d^2/dZ^2)]U_0(Z)+[K_1(d^2/dZ^2)-(i/\pi_1)]W_0(Z)=0 \quad (27)$$

The full solutions ofzero perturbation equations become:

$$U_0(Z)=\frac{(1-K_1)(Z-1)+K_1\,sinh\,(\alpha_0(Z-1))}{(\alpha_0\,cosh(\alpha_0))} \quad (28)$$
$$W_0(Z)=\frac{-\,sinh\,(\alpha_0(Z-1))}{(\alpha_0\,cosh(\alpha_0))} \quad (29)$$
$$P_0(Z)=-K_1[1-\cosh(\alpha_0(Z-1))/cosh(\alpha_0)] \quad (30)$$
$$\sigma_{ZZ_0}(Z)=1$$

Where, $\alpha_0=\sqrt{(i/\pi_1)/[K_1(1-K_1)]}$

From the definition of the perturbation method in equation (15), the first perturbation equations become:

$$(1-K_1)\frac{\partial\sigma_{ZZ_1}(Z,T)}{\partial Z}=\beta\left\{\frac{\partial W_0(Z,T)}{\partial T}\frac{\partial^2 W_0(Z,T)}{\partial Z\,\partial T}\right\} \quad (31)$$
$$-(1-K_1)\frac{\partial P_1(Z,T)}{\partial Z}=\gamma\left\{\frac{\partial W_0(Z,T)}{\partial T}\frac{\partial^2 W_0(Z,T)}{\partial Z\,\partial T}\right\}+\frac{1}{\pi_1\sqrt{\pi_2}}\frac{\partial W_1(Z,T)}{\partial T} \quad (32)$$
$$\frac{\partial\sigma_{ZZ_1}(Z,T)}{\partial T}=\frac{\partial^2 U_1(Z,T)}{\partial Z\,\partial T}+f(Z)\frac{\partial^2 U_0(Z,T)}{\partial Z\,\partial T}-\frac{\partial P_1(Z,T)}{\partial T} \quad (33)$$
$$-\frac{\partial P_1(Z,T)}{\partial T}=\frac{K_1}{1-K_1}\left[\frac{\partial^2 U_1(Z,T)}{\partial Z\,\partial T}+\frac{\partial^2 W_1(Z,T)}{\partial Z\,\partial T}\right] \quad (34)$$

The two equations of firstperturbation will be taking the form:

$$\left[\frac{d^2}{dZ^2}\right]U_1(Z)+\left[K_1\left(\frac{d^2}{dZ^2}\right)\right]W_1(Z)=-(1-K_1)\frac{d}{dZ}\left(f(Z)\frac{dU_0(Z)}{dZ}\right)$$
$$-\beta\pi_2(W_0(Z)\,dW_0(Z)/dZ) \quad (35)$$
$$[K_1(d^2/dZ^2)]U_1(Z)+[K_1(d^2/dZ^2)-(2i/\pi_1)]W_1(Z)=-\gamma\pi_2(W_0(Z)\,dW_0(Z)/dZ) \quad (36)$$

The full solutions of the first perturbationequationsbecome:

Let, $f(Z) = \sin Z$, *and* $\alpha_1 = \sqrt{\frac{\left(\frac{2i}{\pi_1}\right)}{[K_1(1-K_1)]}}$

$$U_1(Z) = A_{11} + A_{12}Z + A_{13}cosh(\alpha_1 Z) + A_{14}sinh(\alpha_1 Z) + B_{1U}\cosh(iZ) + B_{2U}\cosh\big((\alpha_0 + i)Z - \alpha_0\big) + B_{3U}\cosh\big((\alpha_0 - i)Z - \alpha_0\big) + C_{1U}\, sinh\, 2\alpha_0(Z-1) \quad (37)$$

$$W_1(Z) = A_{15} + A_{16}Z + A_{17}cosh(\alpha_1 Z) + A_{18}sinh(\alpha_1 Z) + B_{1W}\cosh(iZ) + B_{2W}\cosh\big((\alpha_0 + i)Z - \alpha_0\big) + B_{3W}\cosh\big((\alpha_0 - i)Z - \alpha_0\big) + C_{1W}\, sinh\, 2\alpha_0(Z-1) \quad (38)$$

$$P_1(Z) = \frac{-K_1}{1-K_1}\{(A_{12} + A_{16}) + \alpha_1\big((A_{13} + A_{17})\, sinh(\alpha_1 Z) + \alpha_1(A_{14} + A_{18})\, cosh(\alpha_1 Z)\big) + (i)(B_{1U} + B_{1W})\, sinh(iZ) + (\alpha_0 + i)(B_{2U} + B_{2W})\, sinh\big((\alpha_0 + i)Z - \alpha_0\big) + (\alpha_0 - i)(B_{3U} + B_{3W})\, sinh\big((\alpha_0 - i)Z - \alpha_0\big) + (2\alpha_0)(C_{1U} + C_{1W})\, cosh\big(2\alpha_0(Z-1)\big)\} \quad (39)$$

$$\sigma_{ZZ_1}(Z) = \frac{1}{1-K_1}\{(A_{12} + K_1A_{16}) + \alpha_1\big((A_{13} + K_1A_{17})\, sinh(\alpha_1 Z) + \alpha_1(A_{14} + K_1A_{18})\, cosh(\alpha_1 Z)\big) + (i)(B_{1U} + K_1B_{1W})\, sinh(iZ) + (\alpha_0 + i)(B_{2U} + K_1B_{2W})\, sinh\big((\alpha_0 + i)Z - \alpha_0\big) + (\alpha_0 - i)(B_{3U} + K_1B_{3W})\, sinh\big((\alpha_0 - i)Z - \alpha_0\big) + (2\alpha_0)(C_{1U} + K_1C_{1W})\, cosh\big(2\alpha_0(Z-1)\big)\} + Sin(Z)\big[1 - K_1 + K_1\, cosh\,\big(\alpha_0(Z-1)\big)/cosh(\alpha_0)\big] \quad (40)$$

Where,

$A_{11} = -A_{12} - A_{13}\cosh\alpha_1 - A_{14}\sinh\alpha_1 + N_1$, $A_{12} = -(N_4/\alpha_1^2) + N_2$

$A_{13} = [-(N_4\sinh\alpha_1/\alpha_1) + N_3]/\alpha_1^2\cosh\alpha_1$, $A_{14} = N_4/\alpha_1^3$

$A_{15} = -A_{16} - A_{17}\cosh\alpha_1 - A_{18}\sinh\alpha_1 + N_5$, $A_{16} = -(N_8/\alpha_1^2) + N_6$

$A_{17} = [-(N_8\sinh\alpha_1/\alpha_1) + N_7]/\alpha_1^2\cosh\alpha_1$, $A_{18} = N_8/\alpha_1^3$

$B_{1U} = a_1 / [K_1(1-K_1)(i)^4 - (2i/\pi_1)(i)^2]$

$B_{2U} = a_2 / [K_1(1-K_1)(\alpha_0 + i)^4 - (2i/\pi_1)(\alpha_0 + i)^2]$

$B_{3U} = a_3 / [K_1(1-K_1)(\alpha_0 - i)^4 - (2i/\pi_1)(\alpha_0 - i)^2]$

$B_{1W} = a_5 / [K_1(1-K_1)(i)^4 - (2i/\pi_1)(i)^2]$

$B_{2W} = a_6 / [K_1(1-K_1)(\alpha_0 + i)^4 - (2i/\pi_1)(\alpha_0 + i)^2]$

$B_{3W} = a_7 / [K_1(1-K_1)(\alpha_0 - i)^4 - (2i/\pi_1)(\alpha_0 - i)^2]$

$C_{1U} = b_1 / [K_1(1-K_1)(2\alpha_0)^4 - (2i/\pi_1)(2\alpha_0)^2]$

$C_{1W} = b_5 / [K_1(1-K_1)(2\alpha_0)^4 - (2i/\pi_1)(2\alpha_0)^2]$

$a_1 = (1-K_1)^2[K_1 + 2i/\pi_1]$

$a_2 = (1 - i\alpha_0)(1-K_1)[-K_1(\alpha_0 + i)^2 + 2i/\pi_1]K_1/2cosh\,\alpha_0$

$a_3 = (1 + i\alpha_0)(1-K_1)[-K_1(\alpha_0 - i)^2 + 2i/\pi_1]K_1/2cosh\,\alpha_0$

$a_5 = K_1(1-K_1)^2$

$a_6 = -K_1(1 - i\alpha_0)(1-K_1)(\alpha_0 + i)^2{K_1}^2/2cosh\,\alpha_0$

$a_7 = -K_1(1 + i\alpha_0)(1-K_1)(\alpha_0 - i)^2{K_1}^2/2cosh\,\alpha_0$

$b_1 = [-K_1\pi_2(\beta - \gamma)(2\alpha_0)^2 + 2\beta\pi_2(i/\pi_1)]/2\alpha_0 cosh\,\alpha_0$

$b_2 = [\pi_2(K_1\beta - \gamma)(2\alpha_0)^2]/2\alpha_0 cosh\,\alpha_0$

$N_1 = -\cosh(i)\,[B_{1U} + B_{2U} + B_{3U}]$

$N_2 = \sinh(\alpha_0)\,[B_{2U}(\alpha_0 + i) + B_{3U}(\alpha_0 - i)] - C_{1U}(2\alpha_0)\cosh(2\alpha_0)$

$N_3 = -\cosh(i)\,[B_{1U}(i)^2 + B_{2U}(\alpha_0 + i)^2 + B_{3U}(\alpha_0 - i)^2] - \cos(1)\,[1 - K_1 + K_1/cosh(\alpha_0)]$

$N_4 = \sinh(\alpha_0)\,[B_{2U}(\alpha_0 + i)^3 + B_{3U}(\alpha_0 - i)^3] - C_{1U}(2\alpha_0)^3\cosh(2\alpha_0) + \pi_2[(\gamma - \beta)/(1-K_1)][1 + tanh^2(\alpha_0)] - 2[K_1\alpha_0\, tanh(\alpha_0)]$

$N_5 = -\cosh(i)\,[B_{1W} + B_{2W} + B_{3W}]$

$N_6 = \sinh(\alpha_0)\,[B_{2W}(\alpha_0 + i) + B_{3W}(\alpha_0 - i)] - C_{1W}(2\alpha_0)\cosh(2\alpha_0)$

$N_7 = -\cosh(i)\,[B_{1W}(i)^2 + B_{2W}(\alpha_0 + i)^2 + B_{3W}(\alpha_0 - i)^2] + \cos(1)\,[1 - K_1 + K_1/cosh(\alpha_0)]$

$N_8 = \sinh(\alpha_0)\,[B_{2W}(\alpha_0 + i)^3 + B_{3W}(\alpha_0 - i)^3] - C_{1W}(2\alpha_0)^3\cosh(2\alpha_0)$

$$-\pi_2[((\gamma-\beta)/(1-K_1))+\gamma/K_1][1+tanh^2(\alpha_0)]+2[K_1\alpha_0\,tanh(\alpha_0)]$$

Substituting equations (*28*) through (30) and (38) through (40), into set of equations (15) yield:

$$U(Z,T)=U_0(Z)*\{1+\zeta.(U_1(Z)/U_0(Z))e^{i\varpi T}\}*e^{i\varpi T} \quad (41)$$

$$W(Z,T)=W_0(Z)*\{1+\zeta.(W_1(Z)/W_0(Z))e^{i\varpi T}\}*e^{i\varpi T} \quad (42)$$

$$P(Z,T)=P_0(Z)*\{1+\zeta.(P_1(Z)/P_0(Z))e^{i\varpi T}\}*e^{i\varpi T} \quad (43)$$

$$\sigma_{ZZ}(Z,T)=\sigma_{ZZ_0}(Z)*\left\{1+\zeta.\left(\sigma_{ZZ_1}(Z)/\sigma_{ZZ_0}(Z)\right)e^{i\varpi T}\right\}*e^{i\varpi T} \quad (44)$$

Result and Discussion

In the following sections, the analysis of the one dimensional fully saturated soil under periodic loads (complete case) are presented and discussed, all results are in good agreement with Zienkiewicz *et al*. (1980, 1999). The distribution of pore pressure, total stress, solid displacement, and fluid displacement with depth are introduced for many values of π_1 andπ_2. Also, a comparison between linear and nonlinear cases is presented (including nonlinear rigidity).The values of parameters used in the results are:

$E=3*10^6\,N/m^2$, $\nu=0.2$, $n=0.3$, $K_s=10*10^9\,N/m^2$, $K_f=4*10^6\,N/m^2$
$Q=120*10^6\,N/m^2$, and $D_0=3.3*10^6\,N/m^2$
$h=10m$, $\beta=0.33$, $\gamma=1.11$, $K_1=0.973$, and $T=500$

Figures 2through5illustrate the behavior of the fluid and solid skeleton displacements with the soil depth. From these figures, it is noticed that, the effect of the nonlinearity is very small due to the absence of the dynamic terms for slow phenomena. The nonlinearity slightly appears at higher values ofπ_1. Generally, the displacement is maximum near the surface and decreases away from the surface up to the base and vanished.

Figures 6 and 7 introduce the behavior of pore pressure with the soil depth for different values ofπ_1 andπ_2. It is clear that, thenonlinearity is affected by higher values ofπ_1. The figures show the pressures are zero at the surface and increase gradually with the depth. It can be noticed that at low values of π_1 the pore pressure is almost constant except at surface it belongs to zero.

Figures 8 and 9introduce the distribution of total stress against the depth. From these figures, it is noticed that, the normalized stress is constant and equal to one for linear analysis. Also, at low values of π_1 the nonlinear term has no effect on the analysis and results of linear and nonlinear are close to each other, while, with increasing the values of π_2 the effect of nonlinearity is noticeable specially at higher values of π_1.

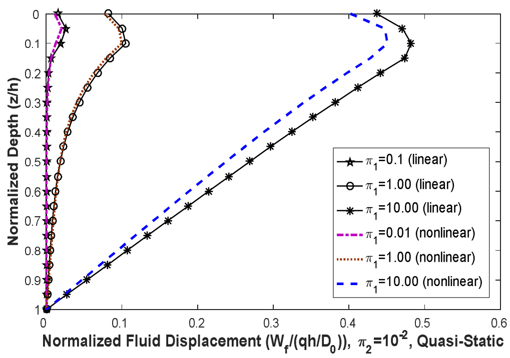

Fig. 2: Fluid displacement with depth,$\pi_2=0.01$

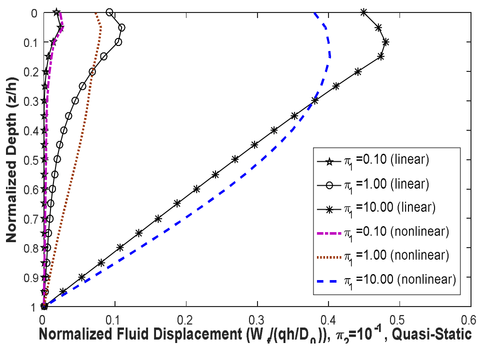

Fig. 3: Fluid displacement with depth,$\pi_2=0.1$

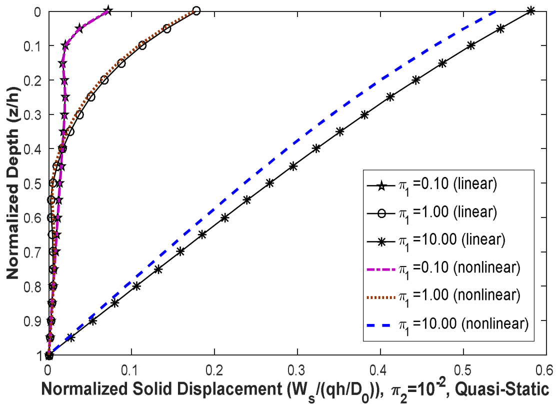

Fig. 4: Solid displacement with depth,π_2 = 0.01

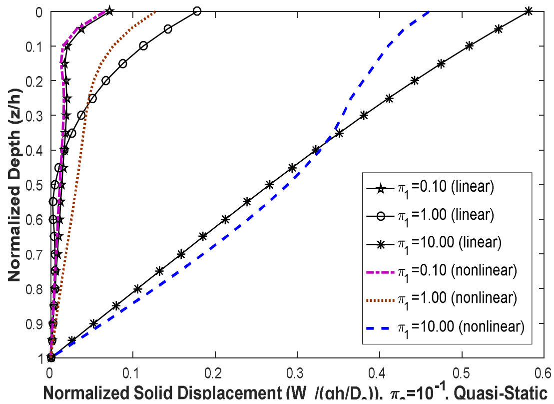

Fig. 5: Solid displacement with depth,π_2 =

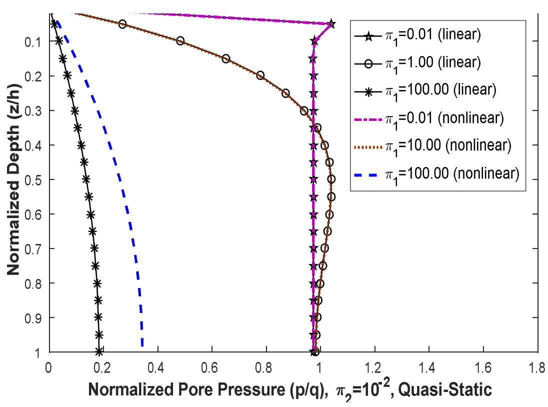

Fig. 6: Pore pressure with depth,π_2 = 0.01

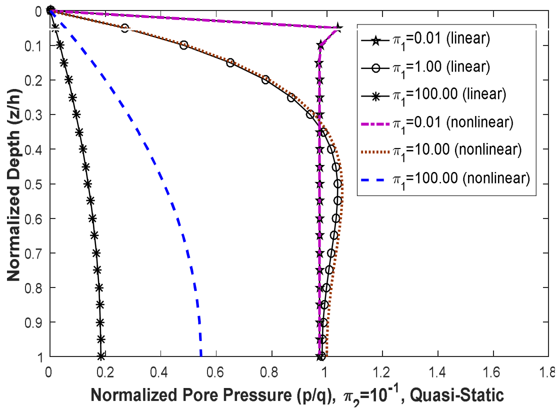

Fig. 7: Pore pressure with depth,π_2 = 0.1

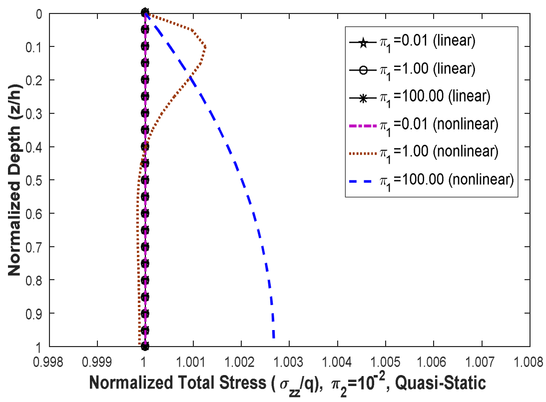

Fig. 8: Total stress with depth,π_2 = 0.01

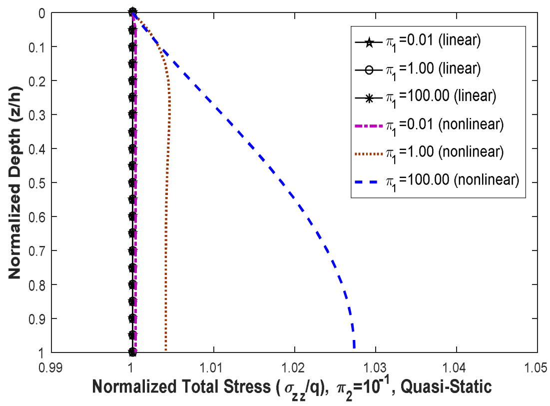

Fig. 9: Total stress with depth,π_2 = 0.1

Conclusion

In this study, a nonlinear analysis of fully saturated porous media is presented by formulate the decoupled nonlinear differential equations of quasi-static consolidation. The governing equations have been non-dimensional and solved analytically using the regular perturbation technique. The effect of nonlinearity of material and geometry on the solid and fluid displacements, the pore pressure, and, total stress was considered. The parametric study of the analysis depends on two dimensionless parameters π_1 (function of permeability) and π_2 (function of frequency of applied load), and It was concluded that:

- At high values of frequencies of applied load, the geometric and material nonlinearity has a great effect on solid and fluid displacements, pore pressure, and total stress especially at higher values of permeability.
- The effect of nonlinearity is very weak due to the absence of the accelerations of solid and fluid for slow phenomena.
- At low level of permeability, almost the geometric nonlinearity has no effect on solid and fluid displacements, pore pressure, and total stress.

References

Attia, H.A., W. Abbas, M.A.M. Abdeen and A.E.D. Abdin, 2014. Effect of porosity on the flow and heat transfer between two parallel porous plates with the Hall effect and variable properties under constant pressure gradient. Blug. Chem. Commun., 46: 535-544.

Attia, H.A., W. Abbas, A.E.D. Abdin and M.A.M. Abdeen, 2015. Effects of Ion Slip and Hall Current on Unsteady Couette Flow of a Dusty Fluid through porous media with Heat Transfer. High Temperature, 53: 891-898.

Attia, H.A., W. Abbas and M.A.M. Abdeen, 2016. Ion slip effect on unsteady Couette flow of a dusty fluid in the presence of uniform suction and injection with heat transfer. Journal of the Brazilian Society of Mechanical Sciences and Engineering, 38: 2381-2391.

Attia, H.A., W. Abbas, A.L. Aboul-Hassan, M.A.M. Abdeen and M.A. Ibrahim, 2016. Unsteady Flow of a Dusty Bingham Fluid through a Porous Medium in a Circular Pipe. Journal of Applied Mechanics and Technical Physics, 57: 596-602.

Boit, M.A., 1941. General theory of three dimensional consolidation. J. Appl. Phys., 12: 155-164.

Boit, M.A., 1956. General solutions of equations of elasticity consolidation for porous material. J. Appl. Mech., ASME, March, pp: 91-96.

Boit, M.A., 1962. Mechanics of deformation and acoustic propagation porous media", *J. Appl. Mech.,* 33: 1482-1498.

Bernhard, A., Schrefler and Roberto Scotta, 2001. A fully coupled dynamic model for two-phase fluid in deformable porous media. Comput. Methods Appl. Mech. Engrg., 3223: 3223-3246.

Elgamal, A., Z. Yang and E. Parra, 2002. Computational modeling of cyclic mobility and post-liquefaction site response. Soil Dynamic and Earthquake Engineering, 22: 259-271.

Elgamal, A., Z. Yang, M. Pastor, E. Parra and A. Ragheb, 2003. Modeling of cyclic mobility in saturated cohesionless soil. Int. J. of Plas.,19(6): 883-905.

Farlow, S.J., 1982. Partial differential equation for scientists and engineers. John Wiley and Sons, Inc.

Hassanen, M. and A. El-Hamalawi, 2007. Two-dimensional development of the dynamic coupled consolidation scaled boundary finite-element method for fully saturated soils. Soil Dynamic and Earthquake Engineering, 27: 153-165.

Heider, Y., O. Avci, B. Markert and W. Ehlers, 2014. The dynamic response of fluid-saturated porous material with application to seismically induced soil liquefaction. Soil Dynamic and Earthquake Engineering, 63: 120-137.

Nedjar, B., 2013. Formulation of a nonlinear porosity law for fully saturated porous media at finite strains. J. of the Mech. And Phys. of solids, 61: 537-556.

Rohan, E., V. Lukes, 2015. "Modeling nonlinear phenomena in deforming fluid-saturated porous media using homogenization and sensitivity analysis concepts", *App. Math. And Comp.*, 267: 583-595.

Roza, A., A. Behzad, 2016. Numerical modeling of subsidence in saturated porous media: Amass conservative method. J. of Hydrology, 542: 423-436.

Truesdell, C., 1957. Sullebasidellatemomeccanica. Rend. Linc., 22: 33-38, 158-166.

Truesdell, C. and R.A. Touppin, 1960. The classical field theories. In S. Flugge (Ed.), Handbuck der Physic Bd III/I, Springer Verlag.

Xikui Li and O.C. Zienkiewicz, 1992. Multiphase flow in deforming porous media and finite element solutions. Comp. & Str., 45(2): 211-227.

Xu, C., J. Song, X. Du and Z. Zhong, 2017. A completely explicit finite element method for solving dynamic u-p equations of fluid-saturated porous media. Soil Dynamic and Earthquake Engineering, 97: 364-376.

Zienkiewicz, O.C., C.T. Chang and P. Bettess, 1980. Drained, and undrained, consolidating and dynamic behavior assumption in soil. Geotechnique, 30: 385-395.

Zienkiewicz, O.C., and P. Bettess, 1980. “Soil and other porous media under transient, dynamic condition. Basic formulation. Scientific papers of Inst. Geotechnical Eng., Wroclaw Techn. Univ. No. 32: 243-254.

Zienkiewicz, O.C., 1982. Basic formulation of static and dynamic behavior of soil and other porous media. in J.B. Martins (Ed.), Numerical Methods in Geomechanics, D. Reidl publishing Co.

Zienkiewicz, O.C. and T. Shiomi, 1984. Dynamic behavior of saturated porous media; The generalized Boit formulation and its numerical solutions. J. of Num. and Analy. Meth. In Geomech., 8: 71-89.

Zienkiewicz, O.C., F.R.S. Chan, C. AH, M. Pastor and D.K. Pau, 1990. Static and dynamic behavior of soil: a rational approach to quantitative solutions. I. Fully saturated problems. Proc. R. Soc. Lond., A 429: 285-309.

Zienkiewicz, O.C., A.H.C. Chan, M. Pastor, BA. Schrefler and T. Shiomi, 1999. “Computational Geomechanics with Special Reference to Earthquake Engineering”, *Wiley: New York,,*ISBN: 0-471-98285-7.

Yonggang, Z., G. Fei, Z. Hongwu and L. Mengkai, 2013. Improved convicted particle domain interpolation method for coupled dynamic analysis of fully saturated porous media involving large deformation. Comput. Methods Appl. Mech. Engrg., 257: 150-163.