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Homotopy Perturbation Method for Solving MHD Nanofluid Flow with Heat and Mass Transfer Considering Chemical Reaction Effect

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ABSTRACT

In this work, a Homotopy Perturbation Solution of magnetohydrodynamic (MHD) model of boundary-layer equations for an electrically conducting Nanofluid flow bounded by an infinite parallel vertical permeable plates is to be introduced. This model is applied to study the mass and heat transfer of an unsteady nanofluid, incompressible flow with suction, internal heat generation, Hall Effect and Chemical Reaction under constant pressure gradient. Considered two types of the nanoparticles namely Silver Ag, and Titanium Dioxide TiO_2 with water used as a base nanofluid. The effects of the material's parameters such as nanoparticles type, nanoparticle volume fraction, heat generation parameter, and chemical reaction parameter on the, velocity, temperature, and concentration are discussed. Results are illustrated graphically for the presented model. It is observed that, the nanoparticles enhance the thermal conductivity of the fluid which implies efficiency improvement of heat transfer systems, and the rate of heat transfer in Ag-water nanofluid is higher than that of Tio_2-water.

Key words: Homotopy Perturbation Method, Heat Transfer, Nanofluid flow, Unsteady flow.

Introduction

Applied mathematical and physical problems in different fields of engineering and science are described by complex systems of nonlinear partial differential equations (Motsa *et al.,* 2012). Analytical methods such as Homotopy Analysis Method (HAM) has been successfully applied to many engineering and science problems, because Homotopy Analysis Method provides us with a convenient way to adjust the convergence rate and the convergence region of the solution series, which is a fundamental qualitative difference in the analysis between HAM and other methods (Omowaye *et al.,* 2015; Liao,1992; Liao, 2003; Bataineh *et al.,* 2009a,b). The homotopy perturbation method (HPM), is a coupling of the traditional perturbation method and homotopy technique for solving various linear, nonlinear initial and boundary value problems, was first proposed by Ji-Huan, (1999&2000) . Recently, it has been implemented by many researchers to find an analytical solution of nonlinear partial differential equations. Shirazpour *et al.* (2011), presented the Homotopy Perturbation Method to obtain the analytical solution of a fully developed flow in a parallel plates channel under the action of Lorentz force. Barik *et al.* (2014), used the Homotopy perturbation method to solve the Couette flow of an incompressible conducting elastico-viscous for Maxwell model fluid in a parallel flat channel. The system of non-linear partial differential equations governing the MHD boundary layer equations with low pressure gradient over a flat plate are solved using Homotopy Perturbation Method by Jhankal (2014).

Heat transfer fluid have wide range of industrial and engineering applications, including in heat transfer of cooling or heating, transport, energy supply, engine cooling, solar energy and electronic cooling, etc. Traditional Heat transfer fluids such as water, and oils have inherently poor heat transfer performance due to their low thermal conductivities. Research and development activities are being carried out to improve the heat transport properties of fluids. The term Nanofluids is defined as a mixture of nanoparticles (nominally 1-100 nm in size) of Solid metallic materials, such as silver,

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copper and iron, and non-metallic materials, such as alumina, CuO, SiC and carbon nanotubes suspended in a base fluid. This term was introduced by Liao (1992) in 1995, he found that the thermal conductivity will be increased if some amount of nanoparticles is added to the base fluid. Since then, many research investigated nanofluids for heat transfer improvement in different thermal applications. Sheikholeslami and Ganji, (2014), studied the three dimensional nanofluid flow of a rotating system with heat transfer and in the presence of a magnetic field. Thermophilic radiative hydromagnetic nanofluid flow with heat and mass transfer over an exponentially stretching porous sheet embedded in porous medium and in the presence of the internal heat generation/absorption, viscous dissipation and suction/injection effects was analyzed by Naramgari and Sulochana, 2016. Anwar *et al.* (2017), investigated the magnetohydrodynamic stagnation-point flow of micropolar nanofluid over a stretching sheet.

Flow with heat and mass transfer problems of Newtonian and non-Newtonian fluid between flat walls is an interesting topic that has been considered by many researchers due to its wide applications in different industries such as electronic cooling, and solar collectors (Attia *et al.,* 2014a&b; Attia *et al.,* 2015). De Haro *et al.*(2014) , analyzed the flow of Heat transfer and entropy generation for non-Newtonian power-law fluid with asymmetric convective cooling between two parallel plates. The unsteady Couette flow of an incompressible, viscous, electrically conducting fluid between two infinite non- conducting porous plates under the boundary layer approximations and presence of both Hall currents and ion-slip was studied by Ghara *et al.* (2012). The study of MHD flows became very important in recent years because of its possible applications in many branches of Science and Technology. There are comprehensive investigations about this topic in references (Tufail *et al.,* 2014; Abdeen *et al.,* 2013; Gireesha *et al.,* 2016; El Kot and Abbas, 2017).

The objective of the present study is to find an analytical solution using homotopy perturbation method (HPM) for an unsteady, MHD Nanofluid, incompressible flow bounded by an infinite parallel vertical permeable plates. We considered two types of the nanoparticles namely Silver Ag, and Titanium Dioxide TiO_2 with water used as a base nanofluid. Influences of first order chemical reaction are also considered. The problem is formulated, solved and pertinent results are discussed in details using graphs. Finally, the effects of physical parameters on temperature and concentration profiles are displayed and analyzed with the help of graphs accompanied by comprehensive discussions.

Mathematical Modeling

Consider the mixed flow of two types of nanofluids of density ρ_{nf} and viscosity μ_{nf} bounded by two vertical impermeable parallel walls in the presence of effects of heat and mass transfer and thermal buoyancy. The flow is assumed to be in the x-direction with y-axis normal to it. The two plates are positioned at y = 0 and y = d. The concentration and the temperature at the lower walls are C_1 and T_1 and at the upper wall are C_2 and T_2, respectively. Also, the fluid is Silver Ag, and Titanium Dioxide TiO_2 with water used as a base nanofluid and the nanoparticles are in thermal equilibrium and there is no slip between them. The thermo-physical properties of the base fluid and different nanoparticles are listed in Table 1 (Mohyud-Din *et al.,* 2016). A constant pressure gradient is imposed in the axial x-direction. All the considered functions will depend on the distance between the plates y and the time t only, except the pressure gradient P. A constant magnetic field is applied and produces induced electric field and A uniform magnetic field is applied perpendicular to the plane of the wall and has a constant magnetic flux density B = (0, B_o,0).

Additionally, we express the magnetic field induced Lorentz force per unit volume as:

$$\vec{F} = \vec{J} \times \vec{B} \quad \text{.....} \quad (1)$$

Where, $\vec{B}$ is the induced magnetic vector, and $\vec{J}$ is the electric current density vector which can be obtained by generalized Ohm's law including Hall current as [22]:

$$\vec{J} = \sigma[\vec{E} + \vec{v} \times \vec{B} - \beta\,(\vec{J} \times \vec{B})] \quad \text{.....} \quad (2)$$

Where, σ is the electric conductivity of the fluid, $\vec{v}$ is the velocity vector, $\vec{v}(y,t) = u(y,t)\vec{i} + v_0\vec{j} + w(y,t)\vec{k}$, $\vec{E}$ is the intensity vector of the electric field, and β is

the Hall factor. By neglecting polarization effect, we get the electric field vector equal zero ($\vec{E}$=0). Equation (1) may be solved in $\vec{J}$ to yield:

$$\vec{F} = \frac{\sigma B_0}{1+m^2}(mu - w)\vec{i} + \frac{\sigma B_0}{1+m^2}(u + mw)\vec{k} \quad \ldots\ldots (3)$$

Where, m is the Hall parameter, $m = \beta\, \sigma B_0$.

Under the usual assumptions, the governing partial differential equations of the conservation of mass, momentum, energy, and concentration for the boundary layer in the presence of magnetic field can be expressed as:

$$\rho_{nf}\left(\frac{\partial u}{\partial t} + v_0\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \mu_{nf}\frac{\partial^2 u}{\partial y^2} + \rho_{nf}\,\beta_{Tnf}\,g\,(T - T_1) + \rho_{nf}\,\beta_{Cnf}\,g\,(C - C_1) - \frac{\sigma_{nf}H_a{}^2}{1+m^2}(u + mw) \quad \ldots\ldots (4)$$

$$\rho_{nf}\left(\frac{\partial w}{\partial t} + v_0\frac{\partial w}{\partial y}\right) = \mu_{nf}\frac{\partial^2 w}{\partial y^2} - \frac{\sigma_{nf}H_a{}^2}{1+m^2}(w - mu) - \frac{\mu_{nf}}{K}w \quad \ldots\ldots (5)$$

$$(\rho c_p)_{nf}\left(\frac{\partial T}{\partial t} + v_0\frac{\partial T}{\partial y}\right) = K_{nf}\frac{\partial^2 T}{\partial y^2} + Q(T - T_1) + \mu_{nf}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] + \frac{\sigma_{nf}H_a{}^2}{1+m^2}(u^2 + w^2) \quad \ldots\ldots (6)$$

$$\frac{\partial C}{\partial t} + v_0\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + K_c(C - C_1) \quad \ldots\ldots (7)$$

Where, g is the gravitational acceleration, β_{Tnf} is the nanofluid coefficient of thermal and expansion due to temperature difference, $\beta_{Cnf}$is the nanofluid coefficient of thermal and expansion due to concentration difference. T is the temperature of the fluid, K_{nf} is the nanofluid thermal diffusivity, C the mass concentration, D_B the diffusion coefficient, Cp_{nf} is the specific heat at constant pressure of nanofluid, Q is the heat source or sink, and k_c is the chemical reaction.

In the present investigation, we assumed the initial and boundary conditions used in solving the problem are:

$$t \leq 0 \quad u = 0; \quad w = 0; \quad T = 0 \quad \ldots\ldots (8a)$$

$$t > 0 \quad u = 0; \quad w = 0; \quad T = T_1\,; \quad C = C_1; \quad y = 0 \quad \ldots\ldots (8b)$$

$$t > 0 \quad u = 0; \quad w = 0; \quad T = T_2;; \quad C = C_2 \quad y = d \quad \ldots\ldots (8c)$$

Nanoparticle Thermo-Physical Properties

In the current study, the thermo-physical properties of the nanofluid were determined using the following relations (Khanafer and Vafai, 2011; Selimefendigil and Öztop,2014) :

The density (ρ_{nf}) and heat capacity ($(\rho c_p)_{nf}$) of the nanofluid, and the electric conductivity (σ_{nf}) are defined as:

$$\rho_{nf} = (1 - \emptyset)\rho_f + \emptyset\rho_p \quad \ldots\ldots (9)$$

$$(\rho c_p)_{nf} = (1 - \emptyset)(\rho c_p)_f + \emptyset(\rho c_p)_p \quad \ldots\ldots (10)$$

$$\sigma_{nf} = (1 - \emptyset)\sigma_f + \emptyset\sigma_p \quad \ldots\ldots (11)$$

According to Brinkman model the viscosity (μ_{nf}) of nanofluid is given as:

$$\mu_{nf} = \frac{\mu_f}{(1-\emptyset)^{2.5}} \quad \ldots\ldots (12)$$

Table 1: Thermo-physical properties of base fluid and nanoparticles Mohyud-Din *et al.,* 2016 .

	ρ (kg/m^3)	c_p $(J/kg\ K)$	k $(W/m\ K)$	$\beta \times 10^{-5}$ $(1/K)$	σ S/m
Base fluid	997.1	4179	0.613	21	5.510^{-6}
Silver (Ag)	10500	235	429	1.89	$62.1\ 10^6$
Titanium Dioxide (TiO_2)	4250	686.2	8.9538	0.9	$2.6\ 10^6$

The thermal conductivity (k_{nf}) is calculated using the Maxell model as:

$$k_{nf} = \frac{k_p + 2k_f - 2\emptyset(k_f - k_p)}{k_p + 2k_f + \emptyset(k_f - k_p)}k_f \quad \ldots\ldots (13)$$

Where, $\emptyset$ is the volume fraction of nanofluid and the subscripts f, nf, and p denotes base fluid, nanofluid and solid particle respectively.

Dimensionless Parameters

Introducing these non-dimensional parameters to the above equations

$$\hat{x} = \frac{x}{d}, \quad \hat{y} = \frac{y}{d}, \quad \hat{z} = \frac{z}{d}, \quad \hat{t} = \frac{t\,\mu_f}{\rho_f\, d}, \quad \hat{u} = \frac{u\,\rho_f\, h}{\mu_f}, \quad \hat{w} = \frac{w\,\rho_f\, h}{\mu_f}, \quad \hat{p} = \frac{\rho\,\rho_f\, h^2}{{\mu_f}^2},$$

$$\hat{\theta} = \frac{T \quad T_1}{T_2 - T_1}, \qquad \hat{\Gamma} = \frac{C \quad C_1}{C_2 - C_1}$$

The governing equations (4 - 7) reduce to the following non-dimensional form after dropping the cap:

$$\frac{\partial u}{\partial t} + s\frac{\partial u}{\partial y} = -J_1\frac{\partial p}{\partial x} + \frac{J_1}{(1-\emptyset)^{2.5}}\frac{\partial^2 u}{\partial y^2} + J_1 J_2 G_r\theta + J_1 J_3 G_c\Gamma - \frac{{H_a}^2}{1+m^2}J_1 J_4(u + mw) \quad \text{.....(14)}$$

$$\frac{\partial w}{\partial t} + s\frac{\partial w}{\partial y} = \frac{J_1}{(1-\emptyset)^{2.5}}\frac{\partial^2 w}{\partial y^2} - \frac{{H_a}^2}{1+m^2}J_1 J_4(w - mu) \quad \text{.....(15)}$$

$$\frac{\partial \theta}{\partial t} + s\frac{\partial \theta}{\partial y} - \frac{J_5 J_6}{P_r}\frac{\partial^2 \theta}{\partial y^2} + Q_R J_6 \theta + \frac{E_c}{(1-\emptyset)^{2.5}}J_6\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] + \frac{E_c {H_a}^2}{1+m^2}J_6 J_4(u^2 + w^2) \quad \text{.....(16)}$$

$$\frac{\partial \Gamma}{\partial t} + s\frac{\partial \Gamma}{\partial y} = \frac{1}{S_c}\frac{\partial^2 \Gamma}{\partial y^2} + \gamma_c\Gamma \quad \text{.....(17)}$$

In the above equations, $S = \frac{v_o\,\rho_f\, h}{\mu_f}$ is the suction parameter, $Ha^2 = \frac{\sigma_f B_o^2 h^2}{\mu_f}$ is the Hartmann number, $Pr = \frac{c_{p_f}\mu_f}{k_f}$ is the Prandtl number, $Ec = \frac{{\mu_f}^2}{{\rho_f}^2 h^2 c_{p_f}(T_2 - T_1)}$ is the Eckert number. $G_r = \frac{\beta_{Tf}\, g\,(T_2 - T_1)\, h^3\, {\rho_f}^2}{{\mu_f}^3}$ is the Grashof number, $G_C = \frac{\beta_{Cf}\, g\,(C_2 - C_1)\, h^3\, {\rho_f}^2}{{\mu_f}^3}$ is the modified Grashof, $S_C = \frac{\mu_f}{\rho_f D_B}$ is the Schmidt number, $\gamma_C = \frac{k_C\,\rho_f\, h^2}{\mu_f}$ is the chemical reaction parameter. $J_1 = \frac{\rho_f}{\rho_{nf}}$, $J_2 = (1-\emptyset) + \emptyset\frac{\rho_s\,\beta_s}{\rho_f\,\beta_f}$, $J_3 = (1-\emptyset) + \emptyset\frac{\rho_s\,\beta_{cs}}{\rho_f\,\beta c_f}$, $J_4 = \frac{\sigma_{nf}}{\sigma_f}$, $J_5 = \frac{k_{nf}}{k_f}$, and $J_6 = \frac{(\rho c_p)_{nf}}{(\rho c_p)_f}$.

Subjected to the boundary conditions:

$$t \le 0 \quad u = 0; \quad w = 0; \quad T = 0 \quad \text{.....(18a)}$$

$$t > 0 \quad u = 0; \quad w = 0; \quad T = 0; \quad C = 0; \quad y = 0 \quad \text{.....(18b)}$$

$$t > 0 \quad u = 0; \quad w = 0; \quad T = 1; \quad C = 1; \quad y = 1 \quad \text{.....(18c)}$$

Homotopy Perturbation Analysis Solutions

In this work, we introduce the homotopy perturbation method (HPM), is considered as a method of analytical solution because of its some advantages over other traditional analytic approximation methods. This method is valid in more general cases of linear and nonlinear problems which does not need a small parameter and provides us with a convenient way to adjust the convergence rate and the convergence region of the solution series.

Basic Idea of Homotopy Perturbation Method

As shown from the above equations that the differential equations are time-dependent, so we present the basic idea of HPM in a way that suits our governing equations. Referring to He's work, the time dependent differential equation has the following general form:

$$A\big(\zeta(r,t)\big) - f(r,t) = 0 \quad r \in \Omega \quad \text{.....(19)}$$

Where, A is a differential operator $\zeta(r,t)$ is an unknown function, r and t denote spatial and temporal independent variables, respectively, $\boldsymbol{\Omega}$ is the domain, and $f(r,t)$ is a known analytic function. Let $f_o(r,t)$ is an initial approximation for equation (19). The differential operator A can be divided into two parts $L(r,t)$ is linear part, and $N(r,t)$ is nonlinear part. Therefore, equation (19) can be rewritten as follows:

$$L(r,t) + N(r,t) - f(r,t) = 0 \quad r \in \boldsymbol{\Omega} \quad \text{.....(20)}$$

Using homotopy technique to construct a homotopy function $\Psi(r,t;p)$ as follows:

$$H(\Psi(r,t;q)) = (1-q)\, L(\Psi(r,t;q)) - L(f_o(r,t)) + qA(\Psi(r,t;q)) - f(r,t) = 0 \quad \text{.....(21)}$$

Where, $q \subset [0,1]$is an embedding parameter.

When q=0 equation (21) becomes:

$$H(\Psi(r,t;0),0) = L(\Psi(r,t;q)) - L(f_o(r,t)) = 0 \quad \text{.....(22)}$$

When q=1 equation (21) becomes:

$$H(\Psi(r,t;1),1) = A(\Psi(r,t;q)) - f(r,t) = 0 \quad \text{.....(23)}$$

Algebraically we have:
$\Psi(r,t;1) = \zeta(r,t)$
$\Psi(r,t;0) = f_o(r,t)$
The changing process of q from zero to unity is just that of $\Psi(r,t;p)$ from $v_o\ (r;\ t)$ to $y(r;\ t)$, this is called deformation. If the embedding parameter $q(0 \leq q \leq 1)$ is considered as a "small parameter", applying the classic perturbation technique. Therefore we can assume that the solution can be represented in a power series form in q as follows:

$$\Psi(r,t;q) = \Psi_o(r,t) + \Psi_1(r,t)\, q + \Psi_2(r,t)\, q^2 + \cdots = \sum_{i=0}^{n} \Psi_i(r,t) q^i \quad \text{.....(24)}$$

Using equation (24) for q=1, one has:

$$\zeta(r,t) = \zeta_o(r,t) + \zeta_1(r,t) + \zeta_2(r,t) + \cdots = \sum_{i=0}^{n} \zeta_i(r,t) \quad \text{.....(25)}$$

Implementation of Homotopy Perturbation Method

According to the homotopy perturbation method to solve the governing equations (14-18), we determine the linear operators which is the linear part of the equation as:

$$L(u(y,t)) = \frac{\partial u}{\partial t} + s\frac{\partial u}{\partial y} + J_1\frac{\partial P}{\partial x} - \frac{J_1}{(1-\emptyset)^{2.5}}\frac{\partial^2 u}{\partial y^2} - J_1 J_2 G_r \theta - J_1 J_3 G_c \Gamma + \frac{H_a{}^2}{1+m^2} J_1 J_4 (u + mw) \quad \text{.....(26)}$$

$$L(w(y,t)) = \frac{\partial w}{\partial t} + s\frac{\partial w}{\partial y} - \frac{J_1}{(1-\emptyset)^{2.5}}\frac{\partial^2 w}{\partial y^2} + \frac{H_a{}^2}{1+m^2} J_1 J_4 (w - mu) \quad \text{.....(27)}$$

$$L(\theta(y,t)) = \frac{\partial \theta}{\partial t} + s\frac{\partial \theta}{\partial y} - \frac{J_5 J_6}{P_r}\frac{\partial^2 \theta}{\partial y^2} - Q_R J_6 \theta - \frac{E_c J_6}{(1-\emptyset)^{2.5}}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] - \frac{E_c H_a{}^2}{1+m^2} J_6 J_4 (u^2 + w^2) \quad \text{.....(28)}$$

$$L(\Gamma(y,t)) = \frac{\partial \Gamma}{\partial t} + s\frac{\partial \Gamma}{\partial y} - \frac{1}{S_c}\frac{\partial^2 \Gamma}{\partial y^2} - \gamma_c \Gamma \quad \text{.....(29)}$$

By applying the homotopy perturbation method structure, the homotopy equations would be constructed as:

$$H(u(y,t;q)) = (1-q)\left(\frac{\partial u}{\partial t} + s\frac{\partial u}{\partial y} + J_1\frac{\partial P}{\partial x} - \frac{J_1}{(1-\phi)^{2.5}}\frac{\partial^2 u}{\partial y^2} - J_1J_2G_r\theta - J_1J_3G_c\Gamma + \frac{H_a{}^2}{1+m^2}j_1J_4(u+mw) - L(u_o)\right) + q\left(\frac{\partial u}{\partial t} + s\frac{\partial u}{\partial y} + J_1\frac{\partial P}{\partial x} - \frac{J_1}{(1-\phi)^{2.5}}\frac{\partial^2 u}{\partial y^2} - J_1J_2G_r\theta - J_1J_3G_c\Gamma + \frac{H_a{}^2}{1+m^2}J_1J_4(u+mw)\right) = 0 \quad \text{.....(30)}$$

$$H(w(y,t;q)) = (1-q)\left(\frac{\partial w}{\partial t} + s\frac{\partial w}{\partial y} - \frac{J_1}{(1-\phi)^{2.5}}\frac{\partial^2 w}{\partial y^2} + \frac{H_a{}^2}{1+m^2}J_1J_4(w-mu) - L(w_o)\right) + q\left(\frac{\partial w}{\partial t} + s\frac{\partial w}{\partial y} - \frac{J_1}{(1-\phi)^{2.5}}\frac{\partial^2 w}{\partial y^2} + \frac{H_a{}^2}{1+m^2}J_1J_4(w-mu)\right) = 0 \quad \text{.....(31)}$$

$$H(\theta(y,t;q)) = (1-q)\left(\frac{\partial \theta}{\partial t} + s\frac{\partial \theta}{\partial y} - \frac{J_5J_6}{P_r}\frac{\partial^2\theta}{\partial y^2} - Q_RJ_6\theta - \frac{E_c}{(1-\phi)^{2.5}}J_6\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] - \frac{E_cH_a{}^2}{1+m^2}J_6J_4(u^2+w^2) - L(\theta_o)\right) + q\left(\frac{\partial \theta}{\partial t} + s\frac{\partial \theta}{\partial y} - \frac{J_5J_6}{P_r}\frac{\partial^2\theta}{\partial y^2} - Q_RJ_6\theta - \frac{E_c}{(1-\phi)^{2.5}}J_6\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] - \frac{E_cH_a{}^2}{1+m^2}J_6J_4(u^2+w^2)\right) = 0 \quad \text{.....(32)}$$

$$H(\Gamma(y,t;q)) = (1-q)\left(\frac{\partial \Gamma}{\partial t} + s\frac{\partial \Gamma}{\partial y} - \frac{1}{S_c}\frac{\partial^2\Gamma}{\partial y^2} - \gamma_c\Gamma - L(\Gamma_o)\right) + q\left(\frac{\partial \Gamma}{\partial t} + s\frac{\partial \Gamma}{\partial y} - \frac{1}{S_c}\frac{\partial^2\Gamma}{\partial y^2} - \gamma_c\Gamma\right) = 0 \quad \text{.....(33)}$$

Implementing the homotopy perturbation method, the solution of equations (30)-(33) can be obtained as follows:

$$u(y,t) = \sum_{n=0}^{\infty} u_n(t)y^n = u_1(t)y + u_2(t)\,y^2 + \cdots$$

$$w(y,t) = \sum_{n=0}^{\infty} w_n(t)y^n = w_1(t)y + w_2(t)\,y^2 + \cdots$$

$$\theta(y,t) = \sum_{n=0}^{\infty} \theta_n(t)y^n = \theta_1(t)y + \theta_2(t)\,y^2 + \cdots$$

$$\Gamma(y,t) = \sum_{n=0}^{\infty} \Gamma_n(t)y^n = \Gamma_1(t)y + \Gamma_2(t)\,y^2 + \cdots$$

Where,

$$u_1(t) = (\mathrm{e}^{-t} - 1)$$

$$u_2(t) = \left(\frac{\frac{-8s}{J_1}\left(1-e^t-\frac{2}{8s}\right) + 2G_C\,J_3 + 2G_r\,J_2 - 2\,J_4\frac{H_a{}^2}{1+m^2} - 3G\,e^t - 6\frac{1}{(1-\phi)^{2.5}}e^t - 2\,G_C\,J_3e^t - 2G_r\,J_2e^t + 2\,J_4\frac{H_a{}^2}{1+m^2}(e^t - m + me^t)}{6a}\right)\mathrm{e}^{-t}$$

$$w_1(t) = (\mathrm{e}^{-t} - 1)$$

$$w_2(t) = \frac{\frac{-8s}{J_1}\left(1-e^t-\frac{2}{8s}\right) + 2\,J_4\frac{H_a{}^2}{1+m^2}(1-e^t + m - me^t)}{6a}\,\mathrm{e}^{-t}$$

$$\theta_1(t) = \mathrm{e}^{-t} - 1$$

$$\theta_2(t) = \frac{\frac{P_r}{J_6}(3s-2s^{-t}-3e^{-t}) + 12J_5 - 6J_5e^{-t} + 2P_rQ_R(1-e^{-t}) + 6E_CP_r\frac{1}{(1-\phi)^{2.5}}(1-2e^{-t}+e^{-2t}) + 3E_CP_rJ_4\frac{H_a{}^2}{1+m^2}(1-2e^{-t}+e^{-2t})}{6J_5}$$

$$\Gamma_1(t) = (1 - \mathrm{e}^{-t})$$

$$\Gamma_2(t) = \frac{-e^{-t}\left(2S_c - 12e^t + 3S_c s + 2\gamma_c S_c - 2\gamma_c S_c e^t - 3S_c s e^t + 6\right)}{6}$$

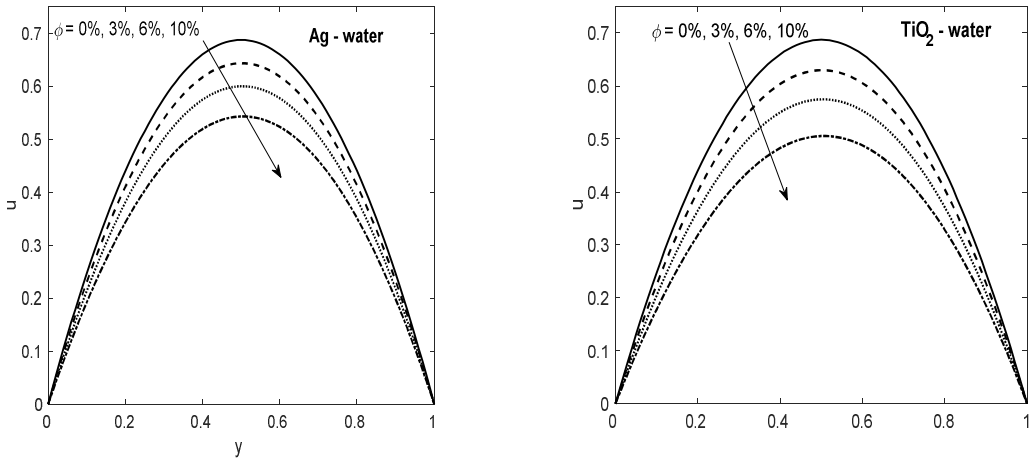
Results and Discussion

In the present study, the base fluid is water containing two different types of the nanoparticles namely silver (Ag), and titanium dioxide (TiO_2). The influence of the material's parameters such as nanoparticle volume fraction $\emptyset$, nanoparticles type, suction parameter s, heat generation/absorption parameter, and chemical reaction parameter γ_c on the velocity and temperature profile within the boundary layer are discussed in details in this section. We have used data related to the thermos-physical properties of the fluid and nanoparticles as listed in table 1.

Effect of nanoparticle volume fraction:

Figure 1, illustrates the effect of nanoparticle volume fraction $\emptyset$ on the velocity profile of the flow for Ag-water, and Tio_2-water nanofluid, respectively. It is observed that, with an increase in the volume fraction the velocity decrease of both Ag-water, and TiO_2-water nanofluid. It is due to the fact that an increase in the volume fraction improves the density of the nanofluid and it causes to slowdown the fluid velocity.

Figure 2, illustrates the effect of nanoparticle volume fractions $\emptyset$ on temperature profile. It is clear that, the amount of heat transfer increase with an increase in the volume fraction $\emptyset$ of the nanoparticles and improves the thermal conductivity. This increase is due to the increase of the surface area of the metallic nanoparticles, this lead to increase the nanolayer around the nanoparticles.

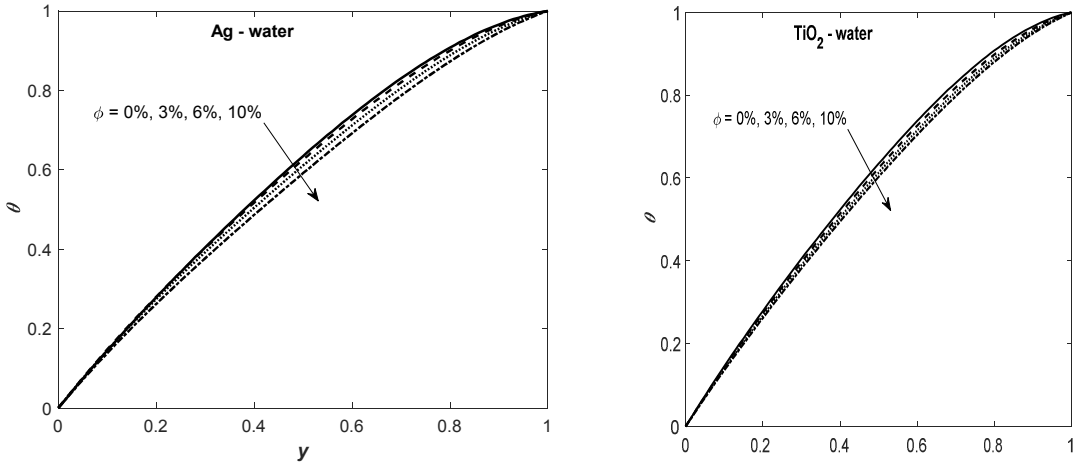


(a) (b)

Fig. 1: Effects of the nanoparticle volume fraction on the velocity profile
(a) Ag-water; (b) TiO_2-water.

Type of nanoparticles:

In figures 1 and 2, represents the influence of different two types of nanoparticles silver, and titanium dioxide on velocity and temperature profiles. It can be seen that the values of the velocity gradient increased gradually by changing the nanoparticle from Tio_2-water to Ag-water nanofluid but the opposite occurs on temperature gradient. Also, the figurers depict Ag-water nanofluid has high thermal conductivity as compared to Tio2-water, so the enhancement of heat transfer is high in Ag-water nanofluid.



(a) (b)

Fig. 2: Effects of the nanoparticle volume fraction on the temperature profile
(a) Ag-water; (b) TiO_2-water.

Effect of heat generation/absorption parameter:

Figures 3 and 4,respectively, illustrate the effect of various values of the heat generation parameter Q > 0 and heat absorption parameter Q < 0 on velocity and temperature profiles for Ag-water, and Tio_2-water nanofluid flow. It is observed that an increase in the heat generation or absorption parameter increases the temperature profiles. It is due to that, the thermal boundary layer thickness decreases as the heat absorption parameter increases while the thermal boundary layer thickness becomes thinner as the heat generation parameter decreases. Also, the velocity profiles of the nanofluid flow decreases with increase in the heat generation or absorption parameter.

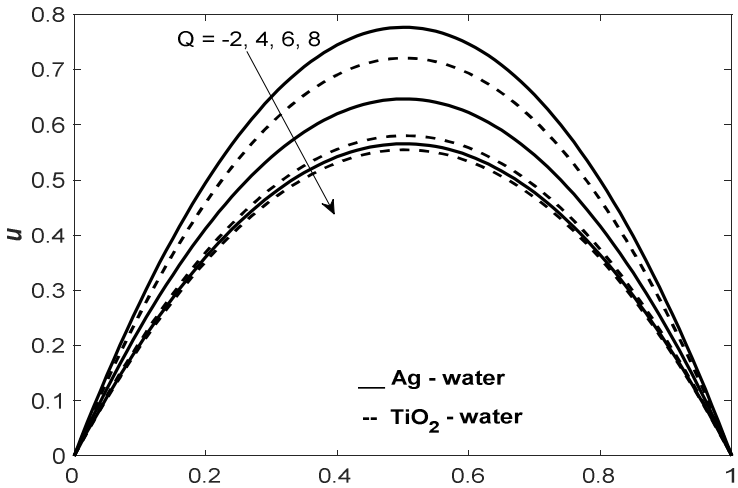


Fig. 3: Effect of heat generation/absorption parameter Q on velocity profile

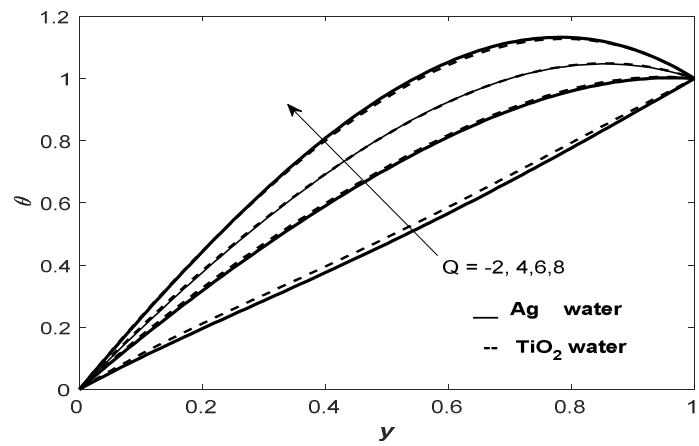


Fig. 4: Effect of heat generation/absorption parameter Q on temperature profile

Effect of suction parameter:

Figures 5 and 6, the influence of suction parameter s on velocity and temperature profiles for Ag-water, and TiO_2-water nanofluid flow. It is observed that, the velocity and temperature decrease as the suction parameter increases of all nanofluid flow. It is noticed that, the thermal boundary layer thickness decrease with increasing in the suction parameter.

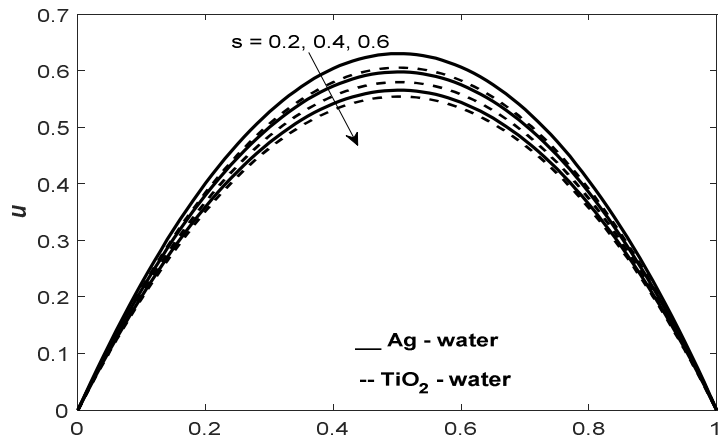


Fig. 5: Effect of suction parameter s on velocity profile.

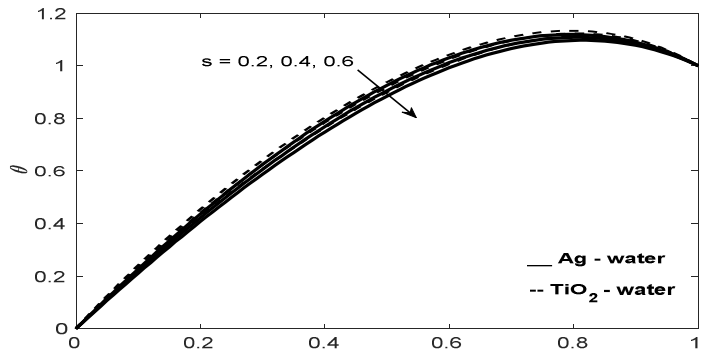


Fig. 6: Effect of suction parameters on temperature profile.

Effect of chemical reaction parameter:

The effect of chemical reaction parameter γ_c on concentration profiles Γ of the Ag-water nanofluid flow are shown in figure7. It is noticed that, the increase of the generating chemical reaction ($\gamma_c > 0$) increases the concentration profile. It is due to the fact that with increasing generation of radioactive solute, the concentration profile is expected to rise, this phenomenon is reversed for consuming chemical reaction ($\gamma_c < 0$).

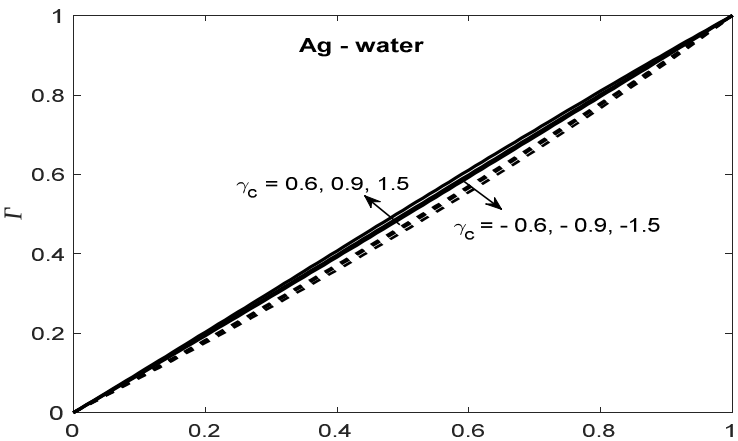


Fig. 7: Effect of chemical reaction parameter γ_c on concentration profile.

Conclusion

This study investigated an analytical solution using Homotopy Perturbation of magnetohydrodynamic (MHD) model of boundary-layer equations for an electrically conducting Nanofluid flow bounded by an infinite parallel vertical permeable plates. The study based on two different types of nanoparticles namely Silver Ag, and Titanium Dioxide TiO_2 with water used as a base nanofluid. The effects of various parameters on velocity, temperature and concentration are discussed with the help of graphs. The findings of the results are summarized as follows:

- Heat transfer increases with the increase in the nanoparticle volume fraction.
- Nanoparticles enhance the thermal conductivity of the fluid which results efficiency improvement of heat transfer systems.
- Ag-water nanofluid enhancement of heat transfer as compared to Tio2-water.
- An increase in the volume fraction improves the density of the nanofluid and decreases the fluid velocity.
- Nanofluid is useful to improve the mechanical properties according to the type and concentration of nanoparticles used.

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